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Social cooperation in Autonomous Agents to Avoid the Tragedy of the commons

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Abstract
In this paper we address the "Tragedy of the Commons" (TOC) problem for shared-resource systems by considering different types of behaviors of agents. On one extreme are self-interested agents while on the other one, agents are concerned about the welfare of the society. Algorithms to capture the different behaviors of the agents with and without interaction among the agents are proposed. An extensive experimental analysis for the different cases has been carried out as well as comparisons of our algorithms with an existing approach. Our study shows that if the agents are willing to sacrifice for some period of time, the sustainability of the society increases considerably.

Keywords
Tragedy of the commons, Cooperation, and Eagerness.

1. Introduction
The "Tragedy of the Commons" (TOC) (López, 2005; Hardin, 2009;Diekert, 2012) is a problem in which the sustainability of the society (group of agents) reduces due to self-interested individual agents using a shared resource (a commons). This problem first appeared in the seminal paper of Hardin in 1968 (Hardin, 2009). Many areas of interest to society like climate change, fisheries management, and preservation of rainforests exhibit this phenomenon (Turner, 1993).

Researchers in the area of Distributed Artificial Intelligence (DAI) and Multi-agent systems (Saha & Sen, 2003; Doebeli & Hauert, 2005; Killingback, Doebeli, & Hauert, 2010; Castelfranchi, 1998;Turner, 1993; Sen & Sen, 2010) have also addressed the TOC problem. In (Turner, 1993), how the TOC is applicable to a DAI setting is studied. In (Saha, 2003) an algorithm is suggested to give optimal resource utilization by the individuals (agents) of the society, where the agents have only local information. In (Sen & Sen, 2010) the performance of the society is studied when aspiration levels are associated with an individual. An aspiration level corresponds to the satisficing return for an individual. Such an aspiration level is adjusted based on past experience.

Any attempt to avoid the tragedy of the commons should incorporate in to the decision making process of an agent the following: the individual gains as well as the social welfare. However these two aspects often conflict. This issue has been addressed in (Hogg & Jennings, 2001) in the context of designing socially intelligent agents, although the TOC problem is not studied in the paper. In (Hogg & Jennings, 2001) a framework is proposed for making socially acceptable decisions.

Consider a society where a public good is available for free (or very little cost) to the members of the society. If there is no law associated for the utilization of the public good, an individual of the society would like to act in a manner that maximizes its utility of the public good. From an individual perspective this is the best decision. However if all individuals act in the like manner, the public good would soon get depleted due to the synergistic behavior and so the society collapses. Thus laws are necessary for the proper functioning of a society. When there is a law in effect it entails a member to abide by it. TOC is concerned with the situation when there is no such formal law or rule. This is where the behavior of an individual comes in to effect that should consider (i) its utility from the public good and (ii) the depletion rate of the public good. If only (i) is considered we are faced with what is called the tragedy of the commons. When (ii) is taken into account the decisions are to some extent based on the welfare of the society.

If we view the public good as a resource, the survival of the society depends directly on the rate of depletion of the resource. The slower the rate of depletion, the longer the time of survival of the society. Although the algorithms developed in (Saha & Sen, 2003; Sen & Sen, 2010) can use the shared resource optimally, the issue of survival time of the society is not considered. In this paper we consider socially motivated agents. The agents make decisions that consider the welfare of the society. This helps the society to survive for a longer period of time compared to the situation when the agents would have acted for their individual gains only.

This paper is organized as follows. In Section 2 we give a model of the TOC problem. We present an algorithm that corresponds to the behavior of self-interested agents and give some experimental results. In Section 3, we present an algorithm that corresponds to the behavior of an agent that is socially motivated and give some experimental results. In Section 4 we define a parameter to quantify an agent's willingness to sacrifice for the society. We present an algorithm that corresponds to socially motivated agents and give some experimental results. In Section 5 we analyze the validity of the results. In Section 6, we present an algorithm that is based on interaction among agents. We conclude in Section 7.

2. Modelling The Tragedy Of The Commons

The tragedy of the commons as developed in Hardin (Hardin, 2009) is as follows:

> "Picture a pasture opens to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. Such an arrangement may work reasonably satisfactorily for centuries because tribal wars, poaching, and disease keep the numbers of both man and beast well below the carrying capacity of the land. Finally, however, comes the day of reckoning, that is, the day when the long-desired goal of social stability

> becomes a reality. At this point, the inherent logic of the commons remorselessly generates tragedy." (Hardin, 2009)

We assume that there are a finite number of agents (herdsmen), are referred to the society. We denote the agents by the numbers 1, 2, …,n. Each agent i has g_i^t number of cattle at time t. Total number of cattle grazing in the field at time t is denoted by G^t. Each cattle consumes Q unit(s) of grass (commons). The field, shared by the cattle of the agents, is available for free to the agents. We refer to the field as the shared resource (initially it is R units).

Resource Available (RA) at time t is denoted as RA^t, (RA^0=R). RA^t denotes the shared resource available to the cattle for consumption and is calculated using (3). All agents increase their cattle from time to time to increase their profit. $incRate_i$ denotes the rate at which an agent i increments its number of cattle. Profit of agent i at time t is denoted by $profit_i^t$ and is calculated using equation (2). Profit of an agent is directly proportional to the number of cattle it uses. Total profit of the society at time t is denoted as $total\,Profit^t$ and is calculated using equation (5). This is the sum of the profits of the agents till the time t. Resource consumed (RC) in time interval ($t-1$, t] is denoted as RC_{t-1}^t. Resource consumed during any time interval is calculated using equation (4)

Initial Conditions: An agent i at time $t = 0$ has g_i^0 number of cattle, $1 \leq i \leq n$. Initial resource available: RA^0=R. We make the following assumptions. The shared resource is non-renewable and quantifiable. Agents do not communicate with each other. No cost is incurred to an agent when it increases the number cattle.

$$G^t = \sum_{i=1}^{n} g_i^t \tag{1}$$

$$profit_i^t = profit_i^{t-1} + Q \times g_i^t \tag{2}$$

$$RA^t = RA^{t-1} - RC_{t-1}^t \tag{3}$$

$$RC_{t-1}^t = \sum_{i=1}^{n} (g_{i\times}^t\ Q) \tag{4}$$

$$totalProfit^t = \sum_{i=1}^{n} \sum_{t=1}^{t} profit_i^t \tag{5}$$

2.1 Goodness Index: Motivation

We wish to define a goodness-index of the society. The following factors influence such an index.

1. The survival time of the society (denoted by K).
2. The total profit received by the society during the survival time.
3. Total number of cattle used during the survival time. It indicates how efficiently the society was able to utilize the resource available.
4. Initial resource available to the society.

We want the society to survive for a longer period of time without compromising much on the total profit. The intuition behind the definition of the index is to obtain a measure of the success of the society that is achieved by utilizing the initial resource available and the number of cattle used. Synergy refers to the interaction of two or more agents so that their combined effect is greater than the sum of their individual effects. It is the combined effect of the society that is of utmost importance even though the agents of the society may not directly interact. Thus we define θ as:

$$\theta = \frac{totalProfit^{k} \times K}{RA^{0\times} G^{K}} \tag{6}$$

We shall use this index to compare the behavior of the agents in different settings. In the following we propose three types of algorithms corresponding to the different behaviors of the agents. In the next subsection we give the first algorithm where we assume that the agents are interested in their individual gains only.

2.2 Algorithm 1: Behavior of Self-interested agents

This algorithm corresponds to the situation where the agents are interested in their individual gains only. The algorithm terminates when the resource gets exhausted. When the algorithm terminates we obtain the total profit and the survival time of the society.

1. $t := 0$
2. While ($RA^t > 0$)
3. for each agent i, $profit_i^t := profit_i^{t-1} + Q \times g_i^t$
4. $totalProfit^t := totalProfit^{t-1} + \sum_{i=1}^{n} profit_i^t$
5. $RC_{t-1}^t := \sum_{i=1}^{n}(g_i^t \times Q)$
6. $RA^t := RA^{t-1} - RC_{t-1}^t$
7. $t := t + 1$
8. for each agent i, $g_i^t := g_i^{t-1} + rand_{0,1} \times incRate_i$

2.3 Experimental Results

For the experiments we took n = 100, Q = 1, RA^0= 10000. The initial number of cattle is obtained as: g_i^t= $rand_{0,1}\times$ 5, where 0 < $rand_{0,1}$ < 1 is a randomly generated value. Increment rate: 0 < $incRate_i \leq 5$.

All the algorithms are implemented using C++11 with gcc version 4.7.2 by including the random header file. We used the pseudo-random number engine Mersenne Twister 19937 generator (class)-mt19937, and uniform real distribution for generating the random numbers[1].

The experimental results for all the algorithms are obtained as follows. For each run, a distinct seed value is assigned to each agent. This corresponds to the different behaviors of the agents. Table 1 shows the seed values for different runs for the agent numbered 10. Consider row 1 of Table 1. The values corresponding to the different attributes are obtained by taking *seed* = 0.827114 for i = 10 and other seed values for all the other agents. Only 5 seed values are shown in Table 1. However we have conducted the experiment for several seed values. The average obtained from these runs is taken to plot the graphs as in Figure 1. Figure 1 shows the results for $incRate_i$ = 5 for all agents. The values obtained are as

[1] Pseudo-random numbers. http://www.cplusplus.com/reference/random/

follows: $totalProfit$ = 10869, survival time is 9 units, total number of cattle used is 2363, and θ= 4.13971 × 10^{-3}.

Table 1. Results for Algorithm 1 for different 'seed' values for an agent

$seed_i$	$g_i^t = 10$	$profit_i^t = 10$	K	G^k	$totalProfit^k$	θ
0.827114	4	137	9	2471	11488	0.00418422
0.525986	3	145	9	2533	11640	0.00413581
0.230025	1	111	9	2510	11619	0.00416618
0.850985	4	112	9	2417	11227	0.00418051
0.728975	3	173	9	2516	11469	0.00410258

From the experimental results we find that the total profit of the society increases until the resource gets exhausted. The survival time of the society decreases on increasing the increment rate.

3. Socially Motivated Agents

In the previous section we considered self-interested agents. Now we consider agents that are socially motivated. That is the agents make decisions that consider the welfare of the society. Unlike the case where an agent was always increasing the cattle, now the agent considers the resource available and the degree to which its profit has been achieved. The motivation being that the decision should be such that helps the society to survive for a longer period of time.

In the previous case we assumed that incrementing the cattle does not incur any cost. Now we associate a cost for purchasing new cattle (X units per cattle).

3.1 Algorithm 2: Behavior of Socially motivated agents

$t := 0$
While ($RA^t > 0$)
for each agent i, $profit_i^t := profit_i^{t-1} + Q \times g_i^t - X \times g_i^t$
$totalProfit^t := totalProfit^{t-1} + \sum_{i=1}^{n} profit_i^t$
$RC_{t-1}^t := \sum_{i=1}^{n} (g_i^t \times Q)$
$RA^t := RA^{t-1} - RC_{t-1}^t$
$t := t + 1$
for each agent i, $g_i^t := g_i^{t-1} + incRate_i()$

Where function $\boldsymbol{incRate_i}()$ is defined as:

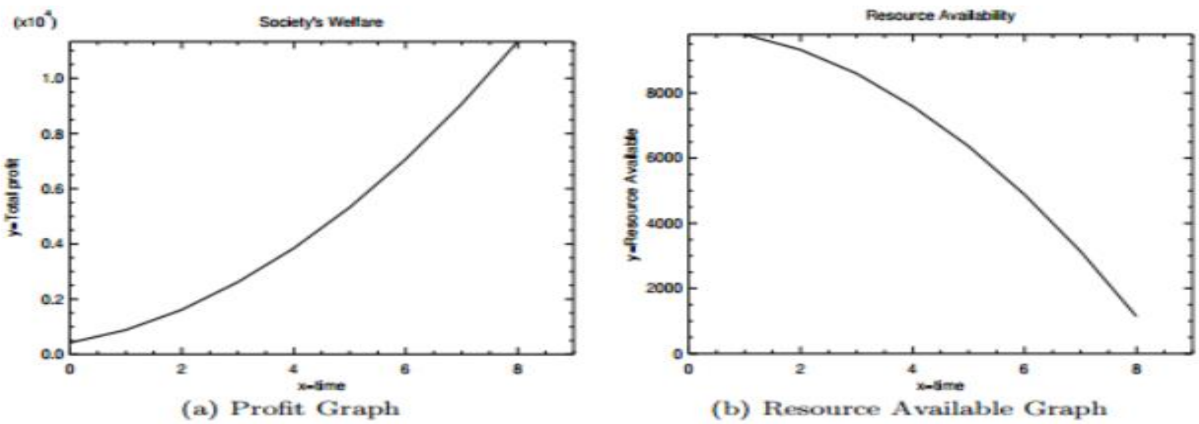


Figure 1. Results for Algorithm 1 where $\boldsymbol{incRate}$ = 5 for all agents

An agent may increase the number of cattle based on (a) the total resource available (case 1), (b) its individual profit (case 2), or (c) combination of both (a) and (b) (case 3).

We assume that shared resource is quantifiable and information about shared resource is available to all agents.

Case 1: The decision is taken based on the total resource available.

if ($RA^t > th_{2,i}$) then
    if ($incRate_i < INC$) then
    $incRate_i$:= $incRate_i$+1
else if ($RA^t \geq th_{1,i}$ and $RA^t < th_{2,i}$) then
    $incRate_i$:= $INC/2$
else if ($RA^t \geq th_{2,i}$)then
    $incRate_i$:= $incRate_i$ ÷1
return $rand_{0,1} \times incRate_i$

Case 2: The decision is taken based only on the individual profit obtained.

if ($profit_i^t > th_{1,i}$) then
$incRate_i$:= $incRate_i$ ÷1
else if ($profit_i^t \geq th_{1,i}$ and $profit_i^t < th_{2,i}$) then
    $incRate_i$:= $incRate_i$-$rand_{0,1} \times \alpha$
else if ($profit_i^t \geq th_{2,i}$) then
    $incRate_i$:= $incRate_i$ ÷2
return $rand_{0,1} \times incRate_i$

Case 3: The decision is taken based on both the resource availability and agent's individual profit.

if ($RA^t > th_{3,i}$) then
    if ($profit_i^t < th_{1,i}$) then
    $incRate_i$:= $incRate_i$+1
    else if ($profit_i^t \geq th_{1,i}$ and $profit_i^t < th_{2,i}$) then
        $incRate_i$:= $incRate_i$-$rand_{0,1} \times \alpha$
    else $incRate_i$:= $incRate_i$ ÷4

else if ($RA^t \geq th_{4,i}$ and $RA^t < th_{3,i}$) then
    if ($profit_i^t < th_{1,i}$) then
        $incRate_i$:= $incRate_i$-1
    else if ($profit_i^t \geq th_{1,i}$ and $profit_i^t < th_{2,i}$) then
        $incRate_i$:= $incRate_i$ ÷2
    else $incRate_i$:= $incRate_i$ ÷4
else if ($RA^t < th_{4,i}$) then
    if ($profit_i^t < th_{1,i}$) then
        $incRate_i$:= $incRate_i$ ÷4
return $rand_{0,1} \times incRate_i$

3.2 Experimental Results

Following results were observed when using the algorithm given in section 3.1. For the experiments we took n = 100, Q = 1, RA^0= 10000. The initial number of cattle is obtained as: g_i^t= $rand_{0,1}$× 5, where 0 < $rand_{0,1}$ < 1 is a randomly generated value. The experimental results are obtained as outlined in section 2.3. Increment rate: $incRate_i$is initialized to 5 for all the experiments. α =2 is the rate at which cattle is decreased.

Table 2 shows the seed values for different runs for the agent numbered 10. The average obtained from these runs is taken to plot the graphs as in Figures 2, 3, and 4.

1. For Case 1, INC =5 is used as the initial maximum increment rate. $th_{1,i} \in (0.5RA^0, 0.65RA^0)$, $th_{2,i} \in (0.85RA^0, 1.0RA^0)$.These are threshold values on the resource available. Figure 2 shows the results for Case 1 of the algorithm. $totalProfit$ = 68096.2, total cattle used = 700, $time$ = 15, and θ = 0.14592.
2. For Case 2, $th_{1,i} = 0.1 \times \frac{RA^0}{n}$, $th_{2,i} = 0.5 \times \frac{RA^0}{n}$. Figure 3 shows the results for Case 2 of the algorithm. $totalProfit$ = 127434, total cattle used = 410, *time* = 24, and θ = 0.777034.
3. For Case 3, $th_{1,i} = 0.1 \times \frac{RA^0}{n}$, $th_{2,i} = 0.5 \times \frac{RA^0}{n}$, $th_{3,i} \in (0.5RA^0, 0.65RA^0)$, $th_{4,i} \in (0.85RA^0, 1.0RA^0)$ Figure 4 shows the results for Case 3 of the algorithm. $totalProfit$ = 84012.6, total cattle used = 564, t*ime* = 18, and θ = 0.268125.

We have used some threshold values on the resource available. These are denoted by $the\ th_{1,i},\ th_{2,i}, th_{3,i},\ and\ th_{4,i}$ values. The values have been obtained experimentally.

3.2 Inference

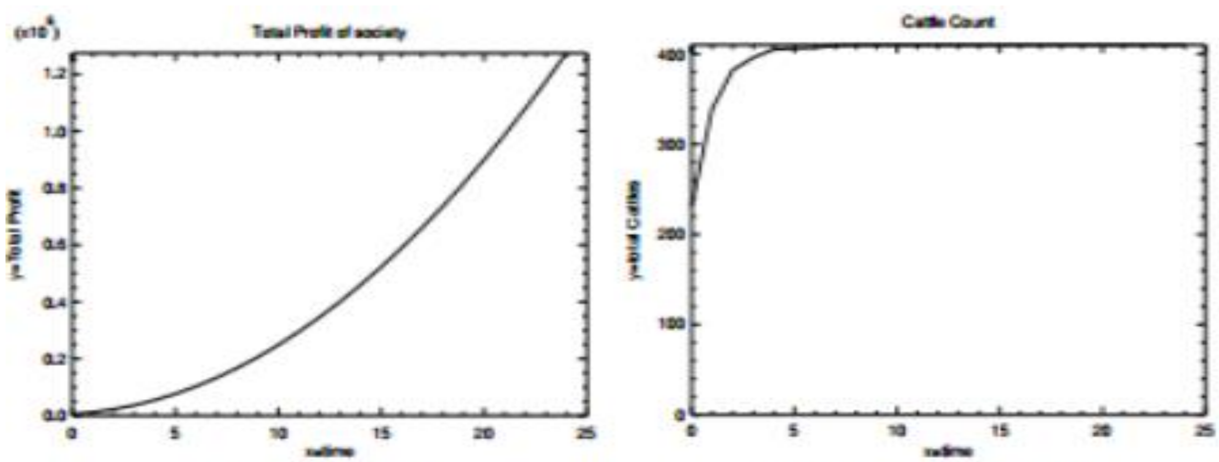
1. The theta values have improved considerably in case 1 as compared to that obtained for the algorithm in section 2.2.
2. In case 2 the agents make decisions based on their individual profits. However, they set their own requirements thresholds and alter the increment rate reasonably, thus increasing the survival time of the society and the θ-value.

3. In case 3, as both the parameters are used for incrementing, the θ-value slightly improves over that for case 1 as the resources quickly get exhausted before any of the agent reaches its second satisfaction level of profits.

From the behavior of the agents given in sections 2.2 and 3.1, we get the idea that if the behavior of all the agents can be controlled in a manner such that they consider the welfare of the society, then at the cost of their personal loss they can contribute to the society that can sustain for a longer period of time. Information about shared resource is hard to calculate. In general, the information is not available to all the agents. Thus agents should be capable of taking decisions based only on their local information.

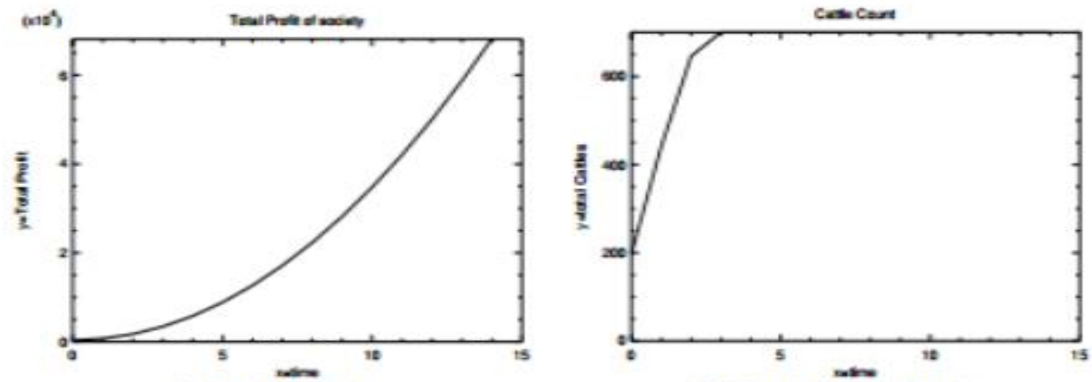
Table 2. Results for Algorithm 2 for different 'seed' values for an agent

$seed_i$	$g_i^t = 10$	$profit_i^t = 10$	K	G^k	$totalProfit^k$	θ
0.702449	3	83	15	689	67930	0.147888
0.957374	4	94	16	679	77612.3	0.182886
0.48069	1	83	23	459	112311	0.562777
0.648213	3	147	22	476	108222	0.500186
0.276817	1	121	22	466	108325	0.511406



(a) Profit Graph (b) Cattle Count Graph

Figure 2. Results for Algorithm 2 : Case 1



(a) Profit Graph (b) Cattle Count Graph

Figure 3. Results for Algorithm 2: Case 2

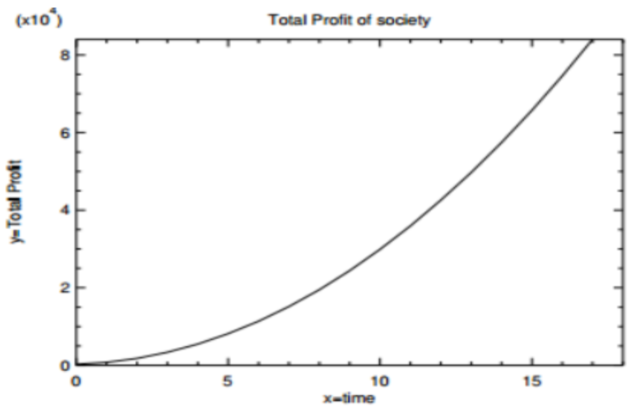


Figure 4. Results for Algorithm 2: Case 3

4. Agent Behavior based on Social Parameter

We consider an agent to have two levels of requirements. The first level (we denote this by $\boldsymbol{need_i}$) reflects the bare minimum requirement that is required for carrying out the minimal operations. The second level (we denote this by $\boldsymbol{greed_i}$) is the aspired greed level. We now introduce a social parameter $\boldsymbol{s_i}$, (a real number whose value lies between 0 and 1) that indicates the willingness of an agent to contribute toward the welfare of the society. Thus an agent may even decrease its cattle at any point of time with a view for enhancing the survival time of the society.

4.1. Algorithm 3: Behavior of Socially Motivated agents based on a social parameter

1. If $need_i$ is not reached then the agent will increase the number of cattle by a random number less than some threshold value.
2. Once the $need_i$ is achieved but less than $greed_i$, the agent will increase its number of cattle with a probability $1-s_i$ and decrease the number of cattle by probability s_i.
3. Once $greed_i$ is achieved the agent first decreases its greed level and then decreases its number of cattle with probability 1.

The pseudo-code for the algorithm is given below:

$t := 0$
While ($RA^t > 0$)
for each agent i, $profit_i^t := profit_i^{t-1} + Q \times g_i^t$
$totalProfit^t := totalProfit^{t-1} + \sum_{i=1}^{n} profit_i^t$
$RC^t := \sum_{i=1}^{n} (g_i^t \times Q)$
$RA^t := RA^{t-1} - RC_{t-1}^t$
$t := t + 1$
for each agent i, $g_i^t := updateCattle(i)$

Where function $\boldsymbol{updateCattle(i)}$ is defined as:

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if ($g_i^t < need_i$) then
  $g_i^t := g_i^t + 1$
else if ($g_i^t \geq need_i$ and $g_i^t < greed_i$) then
  $rv := rand_{0,1}$
  if ($rv < s_i$) then
    $g_i^t := g_i^t \div (1 + s_i)$
  else $g_i^t := g_i^t + 1$
else if ($g_i^t \geq greed_i$) then
  $greed_i$:=($greed_i + need_i$)÷2
$g_i^t := g_i^t \div (1.5 + s_i)$
return g_i^t

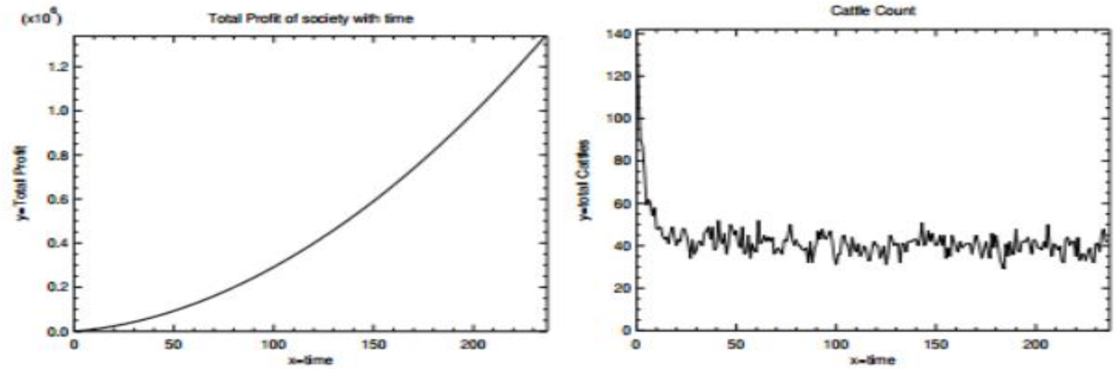
4.2. Experimental Results

For the experiments we took n = 100, Q = 1, RA^0= 10000. The initial number of cattle is obtained as: g_i^t= $rand_{0,1}$× 5, where 0 < $rand_{0,1}$ < 1 is a randomly generated value. The experimental results are obtained as outlined in section 2.3.The initial value of gi is: 0 ≤ g_i ≤ 5. The other values taken for the experiments are: s_i ∈ (0.0, 1.0), $need_i$ ∈ (0.0, 0.01× $\frac{RA^0}{n}$), and $greed_i$ ∈ (0.70× $\frac{RA^0}{n}$, 0.85× $\frac{RA^0}{n}$).

Table 3 shows the seed values for different runs for the agent numbered 10. The average obtained from these runs is taken to plot the graphs as in Figures 5, 6, and 7. Figure 5 shows the results of the algorithm in section 4.1 when s ∈ (0.1, 1.0) $totalProfit$ =1.34E+006, total cattle used= 10038, *time* = 237, θ = 3.1611. Figure 6 and Figure 7 shows the results of the algorithm when s ∈ (0.3, 1.0) $totalProfit$ = 1.49E+ 06, total cattle used is 10015, *time* = 265, θ = 3.9465.

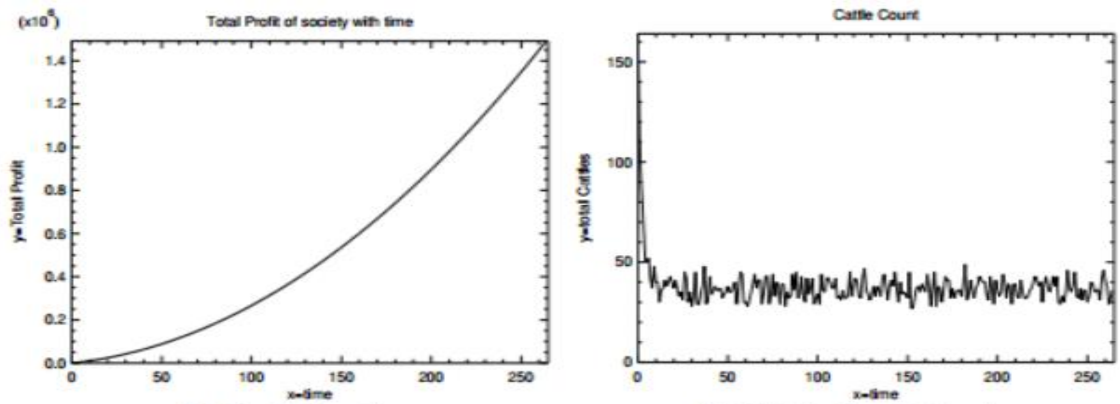
Table 3. Results for Algorithm 3 for different 'seed' values for an agent

$seed_i$	$g_i^t = 10$	$profit_i^t = 10$	K	G^k	$totalProfit^k$	θ
0.935899	3	209	243	10016	1.43E+06	3.46935
0.616328	2	97	262	10038	1.47E+06	3.83682
0.557808	1	100	291	9993	1.58E+06	4.60121
0.6454304	3	110	307	9985	1.66E+06	5.10386
0.0535557	4	61	303	10031	1.64E+06	4.95384



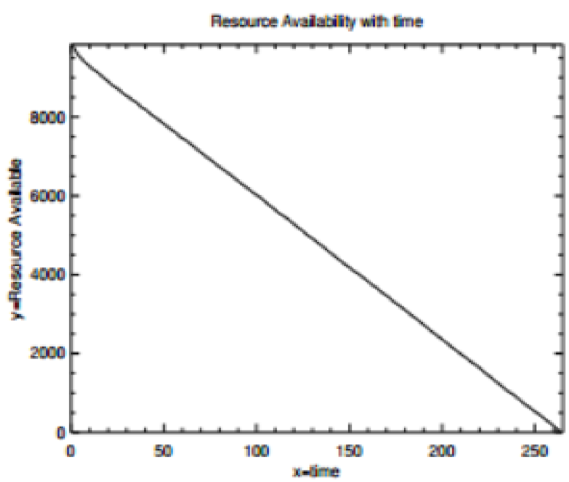
(a) Profit Graph (b) Cattle Count Graph

Figure 5. Results for Algorithm 3 when decision is based on $s \in (0.1,1.0)$



(a) Profit Graph (b) Cattle Count Graph

Figure 6. Results for Algorithm 3 when decision is based on $s \in (0.3,1.0)$



(a) Resource Available Graph

Figure 7. Resource available using Algorithm 3 where decision is based on social parameters $s \in (0.3,1.0)$

When $s \in (0.6, 1.0)$, $totalProfit$ = 1.67E+006, total cattle used is 10002, $time$ = 305, and θ = 5.09486; when $s \in (0.9, 1.0)$ $totalProfit$ = 1.63E+006, total cattle used is 10028, $time$ = 300, and θ = 4.87298.

4.3. Inference

1. The total profit of the society increases quickly up to a threshold value and then varies slightly around it.
2. When the s parameter is increased from (0.1, 1.0) to (0.3, 1.0), the survival time of the society increases from 237 units to 265 units. This shows that if all the agents are ready to sacrifice for at least 30% of the time then the overall survival time of society increases by 11.81% as compared to the situation when all the agents sacrifice for at least 10% of time.
3. On increasing value of s the survival time of the society increases. The value of θ also increases, implying that the society's welfare also increases. But value of θ saturates when all the agents are sacrificing for most of the time.

5. Comparison of the Algorithms

We give an analytical argument of the behavior of the agents (for different cases) based on the parameter θ. Let us first compare the θ values corresponding to Algorithm 1 and Algorithm 2.

$$\theta = \frac{totalProfit^{K_1} \times K_1}{RA^0 \times G^{K_1}} \tag{7}$$

$$\theta_2 = \frac{totalProfit^{K_2} \times K_2}{RA^0 \times G^{K_2}} \tag{8}$$

Now let see the value of $\frac{\theta_2}{\theta_1}$ using equation (7) and (8). So it can be written as:

$$\frac{\theta_2}{\theta_1} = \frac{totalProfit^{K_2}}{totalProfit^{K_1}} \times \frac{K_2}{K_1} \times \frac{G^{K_1}}{G^{K_2}} \tag{9}$$

$$= \left(\frac{totalProfit^{K_2}}{G^{K_2}} \div \frac{totalProfit^{K_1}}{G^{K_1}}\right) \times \frac{K_2}{K_1}$$

$$= \frac{K_2}{K_1}$$

Since total profit is proportional to $\frac{Q \times G^K}{G^K}$, so $\frac{totalProfit^{K_2}}{G^{K_2}}$ gives Q. Hence the ratio of $\frac{totalProfit^{K_2}}{G^{K_2}}$ and $\frac{totalProfit^{K_1}}{G^{K_1}}$ equals to 1. The ratio $\frac{K_2}{K_1} > 1$ because in the second algorithm the number of cattle is a non-decreasing function, while in the first algorithm it is monotonically increasing. Thus, $K_2 > K_1$. So $\frac{\theta_2}{\theta_1} > 1$ and $\theta_2 > \theta_1$. Similarly we compare θ_1 and θ_3 and obtain $\theta_1 < \theta_3$. Now we can compare θ_2 and θ_3 using the above results.

$$\frac{\theta_3}{\theta_2} = \frac{\theta_3}{\theta_1} \div \frac{\theta_2}{\theta_1} \tag{10}$$

$$= \left(\frac{totalProfit^{K_3}}{G^{K_3}} \div \frac{totalProfit^{K_2}}{G^{K_2}}\right) \times \frac{K_3}{K_2}$$

In Algorithm 3 the number of cattle can also decrease while in Algorithm 2 the cattle count is non-decreasing and thus the society will survive for longer period of time in case of Algorithm 3, i.e., $K_3 > K_2$ and thus $\theta_3 > \theta_2$. Combining the above results, we get $\theta_3 > \theta_2 > \theta_1$. Thus, as the actions of the agents consider the welfare of the society, the goodness index of the society increases. This implies that the survival period of the society increases. The summary of the experimental results for the three algorithms is given in Table 4. For Algorithm 3 the θ value is maximum, followed by that for Algorithm 2. The θ value for algorithm 1 is the least. Thus the values obtained from the experiments validate the above relation, i.e., $\theta_3 > \theta_2 > \theta_1$.

Table 4. Comparison of $\boldsymbol{\theta}$ values for the Algorithms

	K	θ
Algorithm 1	9	0.004
Algorithm 2	15 to 23	0.14 to 0.56
Algorithm 3	243 to 307	3.5 to 5.1

6. Social Cooperative approach to avoid the TOC

In the algorithms previously proposed interaction among agents has not been considered. This is motivated by the fact that in many real world scenarios (e.g., too many cars limiting the flow of traffic on a highway **(SENGE, 2014)** or several automobile designers, each making increasing demand on the car battery **(BRAUN, 2002)** the agents simply cannot interact. However if we allow the agents to interact by sharing information, it may be potentially of great benefit for the society. In this section we propose a *socially-cooperative behavior (SMB)* algorithm based on interaction among agents with the ability of maintaining a proper tradeoff between individual gain and social concern. The emergent global society behavior of SMB result in a performance improvement with respect to the non-social behavior algorithms

Similarly to the criteria proposed in Algorithm 3, we consider that each agent has two levels of requirements, i.e., $need_i$ and $greed_i$ where each agent is associated with a *social parameter* s_i. Thus, an agent may contribute to the society by adjusting its load according to its degree of willingness (which refers to the social parameter).

However, for this algorithm, we assume a mathematical model for the TOC which is slightly different from the model in Section 4. Here, the profit of an agent at time $\boldsymbol{t}$ is computed using equation (11)

$$profit^t = \begin{cases} \delta, & RC^t \le R \\ \delta \times \frac{R}{RC^t}, & RC^t > R \end{cases} \tag{11}$$

The rationale behind this model of the TOC is as follows. The first obvious manifestation of equation (11) is that when applied load is less than the capacity of resource, it is pure gain for agent. On the other hand, it is also evident from equation (11) that when an agent makes a selfish choice, profit for each degrades. In such situation, the cooperation between agents can avoid the ruination of the public resource.

Initial conditions: To design the social cooperative behavior algorithm *SCB* to avoid the TOC, the initial conditions are given below.

- $\boldsymbol{need_{total}}$=$\sum_{i=1}^{n} \boldsymbol{need_i}$; total minimum requirement
- $\boldsymbol{greed_i}$: greed of an agent $\boldsymbol{i}$ and is calculated as

$$\boldsymbol{greed_i = greed_{total} \times \frac{(1 - s_i)}{\sum_{i=1}^{n}(1 - s_i)}} \tag{12}$$

  where $\boldsymbol{greed_{total}}$ denotes the total greediness of the system and evaluated as:

$$\boldsymbol{greed_{total} = R - \frac{need_{total}}{2}} \tag{13}$$

- $\boldsymbol{RC_i^t}$ represents total load that agent $\boldsymbol{i}$ willing to put on the shared resource and determined as:

$$\boldsymbol{RC_i^t = need_i + greed_i} \tag{14}$$

A step-by-step procedure for the social cooperative behaviour algorithm is elucidated in Algorithm 4.

6.1. Algorithm 4: social cooperative behavior algorithm

1. Initially each agent i consumes a random amount of resource RC_i^0 from shared resource. Note that RC_i^0 is selected from the interval $(1, need_i)$ and $RC = \sum_{i=1}^{n} RC_i^0$. Thereafter, each agent executes following steps independently.
2. **if** $RC_i < need_i$ **then** /* *agent i checks whether its minimum requirement is fulfilled*
3. **if** $RC < R$ **then** /* *total load on the system is less the resource capacity*
4. Increment the agent increments its consumption by one, i.e., $RC_i := RC_i^0 + 1$
5. $RC := RC + 1$ /* *increase the load consumption by one unit*
6. **else if** $RC \ge R$ then agent i do following
7. $\boldsymbol{Send}$ (*UpdateGreed, j*) to all other agents
8. After receiving $Send$ **(*RC, i*) goto** step 3
9. **else if** $need_i < RC_i < need_i + greed_i$ **then**
10. **goto** step 3

After receiving the message $\boldsymbol{Send}$ **(***UpdateGreed, j***)**, agent $\boldsymbol{j}$ do the following

1. **if** social parameter of agent *j* is less than or equal to the threshold value **then**

If $\boldsymbol{RC_j > need_j + greed_j}$ **then** */* load consumed by agent j exceeds its total requirement*
$\boldsymbol{RC \coloneqq RC - RC_j}$
$\boldsymbol{s_j}$:= $\boldsymbol{s_j + (1 - s_j)/2}$ */* update the social parameter of agent j*
$\boldsymbol{greed_j}$:= $\boldsymbol{greed_j \times (1 - s_j)}$ */* update the greed of agent j*
$\boldsymbol{RC_j}$:= $\boldsymbol{need_j + greed_j}$ */* update the current load consumption of agent j*
$\boldsymbol{RC \coloneqq RC + RC_j}$ */* update the total load consumption on the resource*
$\boldsymbol{Send}$ $(\boldsymbol{RC, i})$ to all other agents

---

6.2. Experimental results

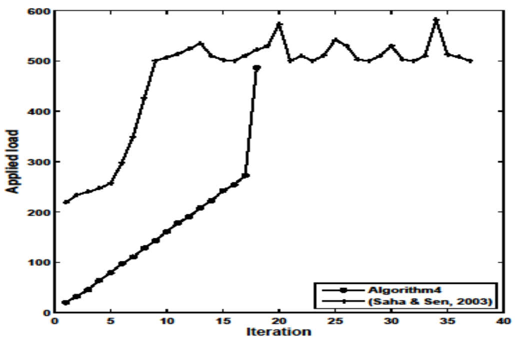
In this section, to analyze the relative performance of the social cooperative algorithm, we compare it against another technique suggested in **(Saha & Sen, 2003)** over varying parametric conditions. The parameters' setting for the experimental study is as follows: both the algorithms are investigated on four different conditions: (i) Number of agents in the society ($\boldsymbol{n}$) is 10 and the capacity of the shared resource ($\boldsymbol{R}$) is 500, (ii) $\boldsymbol{n}$ =20; $\boldsymbol{R}$ =500, (iii) $\boldsymbol{n}$ =40; $\boldsymbol{R}$ =1000, and (iv) $\boldsymbol{n}$ =80; $\boldsymbol{R}$ =1000. Here, the value of $\boldsymbol{\delta}$ is taken as 1 and for all the experiments, the threshold value of the social parameter is 0.9.

The graphical representation of the variation of load in both the algorithms is demonstrated in Figure 8. Figure 8 shows how the agents attain the equilibrium. It is clear from Figure 8(a)-8(d) that after a random initial load, agents steadily increase the load on the shared resource. But, it is interesting to see that our proposed algorithm outperforms its competitor **(Saha & Sen, 2003)** in a statistically significant manner. First, our proposed algorithm achieves equilibrium without making over utilization of resource, while in **(Saha & Sen, 2003)**, there is the over utilization of resource before achieving equilibrium. Second, Algorithm 4 requires comparatively less time (number of iterations) to reach equilibrium than that for the **(Saha & Sen, 2003)** .

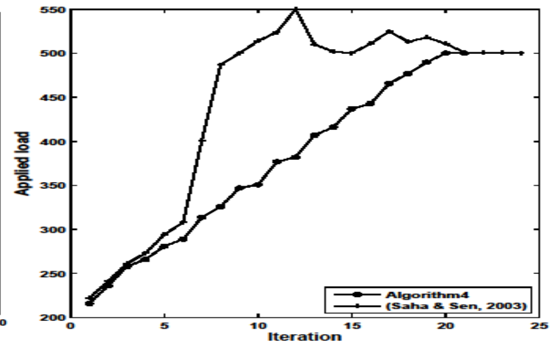
The variation of the average per unit profit of an agent is shown in Figure 9. It is a clear manifestation from Figure 9(a)-9(d) that in Algorithm 4, agent receives a constant profit in the entire time span. It signifies that there is no temptation for agents to free ride on the efforts of others since agents are socially motivated and is disposed to avoid over utilization of resource. On the contrary, in **(Saha & Sen, 2003)**, there is variation in the profit (utility). Initially, each agent attempts to maximize the utility, but when load exceeds the capacity, per unit profit degrades.

6.3. Inference

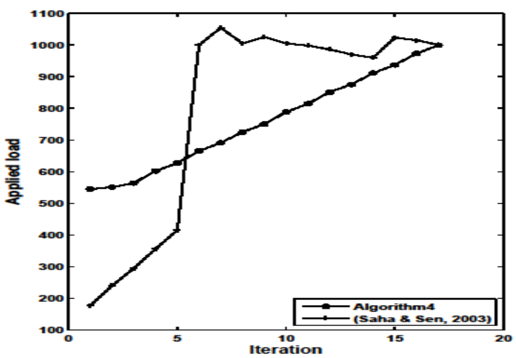
The overall experimental result suggested that that our proposed social cooperation algorithm is capable to avoid the TOC problem. Some of the noticeable features of our proposed decentralized algorithm can be characterized as follows: it quickly converges to equilibrium and it takes into consideration both individual gains as well as societal concern.



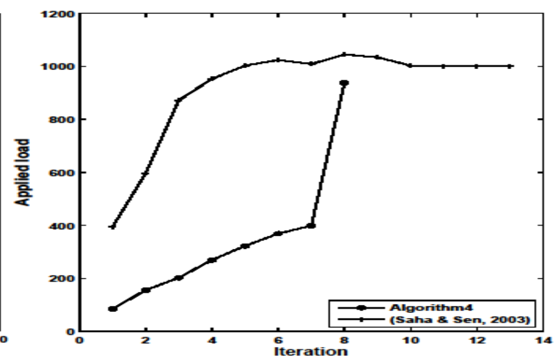
(a) $R = 500, n = 10$



(b) $R = 500, n = 20$

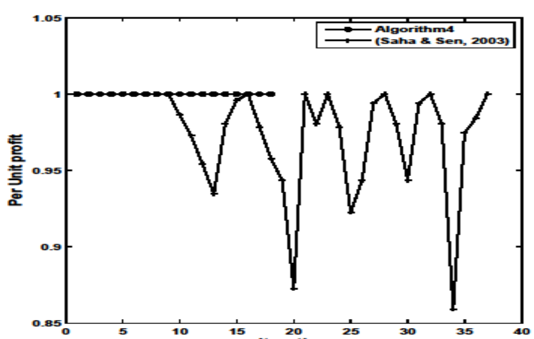


(c) $R = 1000, n = 40$

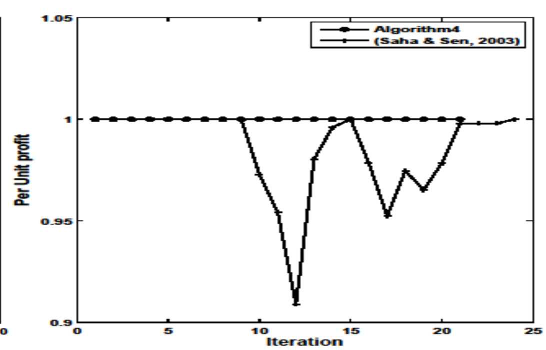


(d) $R = 1000, n = 80$

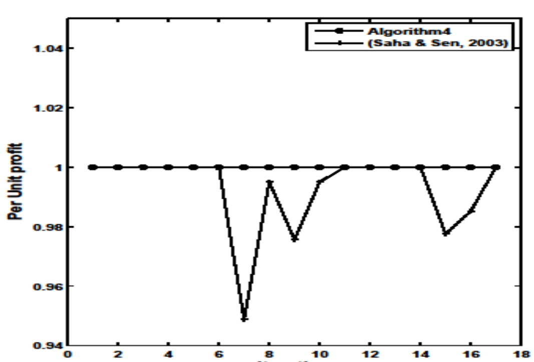
Figure 8: Illustrating the variation of load over the shared resource



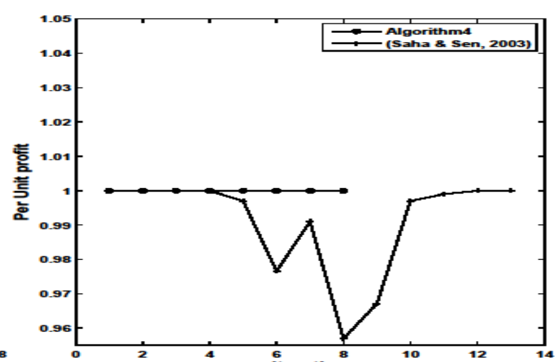
(a) $R = 500, n = 10$



(b) $R = 500, n = 20$



(c) $R = 1000, n = 40$



(d) $R = 1000, n = 80$

Figure 9: Illustrating the average per unit profit

7. Conclusion

In this paper we gave a mathematical modeling of the tragedy of commons problem in shared-resource system. The approach is based on the study of different agent behaviors. First self-interested agents, i.e. the agents interested in their individual gains only, have been considered. Then we assumed that the agents make decisions based on the resource availability, individual gains or combination of both. The second class of agent's behavior takes into account the welfare of the society, then the society survives for a longer time compared to that in previous case. In the third type of behavior considered, we defined a measure of greediness and the agents make decision based on this parameter. It has been observed, on the basis of extensive experiments that society as a whole performs much better with a greediness bounded behavior with respect to a totally self-interest or a totally altruistic behavior. From our study we conclude that if the agents are willing to sacrifice for some period of time, the global sustainability of the society increases considerably. The first part of the study has been held considering mo interaction among the agents, i.e. the autonomous distributed agents do not directly communicate each other, although they can locally observe the common environment they modify. In the second part interaction and cooperation among agents has been considered. A social cooperative behavior (SCB) algorithm has been proposed, that converges to equilibrium quickly and with no potential danger of over utilization of common resources. The proposed socially motivated cooperating agents, strongly rely on the concept of reputation of agents (Milinski, Semmann, & Krambeck, 2002), as the key element which contribute to solve the tragedy of the commons. Experiments shows that the algorithm outperforms state of the art existing approach (Saha & Sen, 2003).

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