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NEARLY CLOAKING THE ELECTROMAGNETIC FIELDS*

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Abstract. The approximate cloaking is investigated for time-harmonic Maxwell's equations via the approach of transformation optics. The problem is reduced to certain boundary effect estimates due to an inhomogeneous electromagnetic inclusion with an asymptotically small support but an arbitrary content enclosed by a thin high-conducting layer. Sharp estimates are established in terms of the asymptotic parameter, which are independent of the material tensors of the small electromagnetic inclusion. The result implies that the "blow-up-a-small-region" construction via the transformation optics approach yields a near-cloak for the electromagnetic waves. A novelty lies in the fact that the geometry of the cloaking construction of this work can be very general. Moreover, by incorporating the conducting layer developed in the present paper right between the cloaked region and the cloaking region, arbitrary electromagnetic contents can be nearly cloaked. Our mathematical technique extends the general one developed in [H. Y. Liu and H. Sun, *J. Math. Pures Appl.*, 99 (2013), pp. 17–42] for nearly cloaking scalar optics. In order to investigate the approximate electromagnetic cloaking for general geometries with arbitrary cloaked contents in the present study, new techniques and analysis tools are developed for this more challenging vector optics case.

Key words. Maxwell's equations, invisibility cloaking, transformation optics, asymptotic estimates, layer potential technique

AMS subject classifications. 35Q60, 35J05, 31B10, 35R30, 78A40

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1. Introduction and statement of the main result. This paper is concerned with invisibility cloaking for electromagnetic (EM) waves via the approach of transformation optics [25, 37], which is a rapidly growing scientific field with many potential applications. We refer to [8, 16, 17, 34, 40, 42] and the references therein for discussions of the recent progress on both the theory and experiments.

Let D and Ω be two bounded smooth domains in $\mathbb{R}^3$, and $D \Subset \Omega$ with D and Ω simply connected, and D containing the origin. Denote

$$D_\rho := \{\rho x;\ x \in D\} \quad \text{for } \rho \in \mathbb{R}_+.$$

Let $\varepsilon(x) = (\varepsilon^{ij}(x))_{i,j=1}^3$, $\mu(x) = (\mu^{ij}(x))_{i,j=1}^3$, and $\sigma(x) = (\sigma^{ij}(x))_{i,j=1}^3$, $x \in \Omega$, be real symmetric-matrix-valued functions, which are bounded in the sense that

$$c|\xi|^2 \le \sum_{i,j=1}^3 \varepsilon^{ij}(x)\xi_i\xi_j \le C|\xi|^2, \quad c|\xi|^2 \le \sum_{i,j=1}^3 \mu^{ij}(x)\xi_i\xi_j \le C|\xi|^2 \tag{1.1}$$

and

$$0 \le \sum_{i,j=1}^3 \sigma^{ij}(x)\xi_i\xi_j \le C|\xi|^2 \tag{1.2}$$

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for all $x \in \Omega$ and $\xi = (\xi_i)_{i=1}^3 \in \mathbb{R}^3$. Here c and C are two generic positive constants whose meanings should be clear from the contexts. Physically, the functions ε, μ, and σ, respectively, stand for the electric permittivity, magnetic permeability, and conductivity tensors of a *regular* EM medium occupying Ω.

Let $0 < \rho < 1$ be a small parameter. Assume that there exists an orientation-preserving bi-Lipschitz mapping $F_\rho : \overline{\Omega}\backslash D_\rho \to \overline{\Omega}\backslash D$, such that

$$F_\rho(\overline{\Omega}\backslash D_\rho) = \overline{\Omega}\backslash D, \qquad F_\rho|_{\partial\Omega} = \text{Identity}. \tag{1.3}$$

Set

$$F(x) = \begin{cases} F_\rho(x), & x \in \Omega\backslash\overline{D}_\rho, \\ \frac{x}{\rho}, & x \in D_\rho. \end{cases} \tag{1.4}$$

Define an EM medium inside $\Omega\backslash\overline{D}$ as

$$\varepsilon_c^\rho(x) = F_*\varepsilon_0(x), \quad \mu_c^\rho(x) = F_*\mu_0(x), \quad \sigma_c^\rho(x) = 0, \quad x \in \Omega\backslash\overline{D}, \tag{1.5}$$

where $\varepsilon_0^{ij} = \delta^{ij}$ and $\mu_0^{ij} = \delta^{ij}$ with δ^{ij} the Kronecker delta function denote the EM parameter tensors of the homogeneous free space. The *push-forward* in (1.5) is defined by

$$F_*m(x) := \left.\frac{DF(y)\cdot m(y)\cdot DF(y)^T}{|\det(DF)(y)|}\right|_{y=F^{-1}(x)}, \quad x \in \Omega\backslash\overline{D}, \tag{1.6}$$

where $m(y), y \in \Omega\backslash\overline{D}_\rho$, denotes an EM parameter in $\Omega\backslash\overline{D}_\rho$, such as ε, μ, or σ, and DF represents the Jacobian matrix of the transformation F. Also, one may rewrite (1.5) as

$$(\Omega\backslash\overline{D}; \varepsilon_c^\rho, \mu_c^\rho) = F_*(\Omega\backslash\overline{D}_\rho; \varepsilon_0, \mu_0) = (F(\Omega\backslash\overline{D}_\rho); F_*\varepsilon_0, F_*\mu_0).$$

In a similar manner, one can define the push-forward of EM parameters from D_ρ to D by replacing $\Omega\backslash\overline{D}_\rho$ and $\Omega\backslash\overline{D}$ in (1.6), respectively, by D_ρ and D. Set

$$(D\backslash\overline{D}_{1/2}; \varepsilon_l, \mu_l, \sigma_l) = F_*(D_\rho\backslash\overline{D}_{\rho/2}; \alpha_0\varepsilon_0, \beta_0\mu_0, \gamma_0\rho^{-2}\delta), \tag{1.7}$$

where α_0, β_0, and γ_0 are positive constants, and

$$(D_{1/2}; \tilde{\varepsilon}_a, \tilde{\mu}_a, \tilde{\sigma}_a) = F_*(D_{\rho/2}; \varepsilon_a, \mu_a, \sigma_a) \tag{1.8}$$

with *arbitrary* but regular ε_a, μ_a, and σ_a. Hence, we have an EM medium in Ω given by

$$\Omega; \tilde{\varepsilon}, \tilde{\mu}, \tilde{\sigma} = \begin{cases} \varepsilon_c^\rho, \mu_c^\rho, \sigma_c^\rho & \text{in } \Omega\backslash\overline{D}, \\ \varepsilon_l, \mu_l, \sigma_l & \text{in } D\backslash\overline{D}_{1/2}, \\ \tilde{\varepsilon}_a, \tilde{\mu}_a, \tilde{\sigma}_a & \text{in } D_{1/2}. \end{cases} \tag{1.9}$$

The time-harmonic EM waves propagating in Ω is governed by the following Maxwell equations:

$$\left\{ \begin{array}{l} \nabla \wedge \widetilde{E}_\rho - i\omega\tilde{\mu}\widetilde{H}_\rho = 0 \\ \nabla \wedge \widetilde{H}_\rho + i\omega\left(\tilde{\varepsilon} + i\dfrac{\tilde{\sigma}}{\omega}\right)\widetilde{E}_\rho = 0 \end{array} \right. \quad \text{in } \Omega, \tag{1.10}$$

where $\widetilde{E}_\rho \in \mathbb{C}^3$ and $\widetilde{H}_\rho \in \mathbb{C}^3$ denote, respectively, the electric and magnetic fields, and $\omega \in \mathbb{R}_+$ denotes the frequency.

Introduce the boundary operator $\widetilde{\Lambda}_\rho$ which maps the tangential component of $\widetilde{E}_\rho|_{\partial\Omega}$ to that of $\widetilde{H}_\rho|_{\partial\Omega}$, i.e.,

$$\widetilde{\Lambda}_\rho(\nu \wedge \widetilde{E}_\rho|_{\partial\Omega}) = \nu \wedge \widetilde{H}_\rho|_{\partial\Omega} : TH_{\mathrm{Div}}^{-1/2}(\partial\Omega) \to TH_{\mathrm{Div}}^{-1/2}(\partial\Omega), \tag{1.11}$$

where ν denotes the outward unit normal vector to $\partial\Omega$, and

$$TH_{\mathrm{Div}}^{-1/2}(\partial\Omega) = \left\{ U \in TH^{-1/2}(\partial\Omega) |\ \mathrm{Div}(U) \in H^{-1/2}(\partial\Omega) \right\}$$

with Div the surface divergence operator on $\partial\Omega$, $TH^s(\partial\Omega)$ the subspace of all those $V \in (H^s(\partial\Omega))^3$ which are orthogonal to ν, and $H^s(\cdot)$ the usual L^2-based Sobolev space of order $s \in \mathbb{R}$. Note that if Γ is the smooth boundary of a bounded domain in $\mathbb{R}^3$, then $H^s(\Gamma)$ and hence $TH^s(\Gamma)$ is well-defined for $|s| \le 2$; see [20] and [27]. In (1.11), $\widetilde{E}_\rho \in H(\nabla\wedge;\Omega)$ is the unique solution to the Maxwell equations (1.10) associated with the boundary condition

$$\nu \wedge \widetilde{E}_\rho|_{\partial\Omega} = \psi \in TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)\ , \tag{1.12}$$

where

$$H(\nabla\wedge;\Omega) = \{U \in (L^2(\Omega))^3 | \nabla \wedge U \in (L^2(\Omega))^3\}.$$

In fact, $\widetilde{\Lambda}_\rho$ is also known as the *admittance map* in the literature.

We further introduce the "free-space" admittance map as follows. Let $E_0 \in H(\nabla\wedge;\Omega)$ and $H_0 \in H(\nabla\wedge;\Omega)$ be solutions to

$$\begin{cases} \nabla \wedge E_0 - i\omega\mu_0 H_0 = 0 & \text{in } \Omega, \\ \nabla \wedge H_0 + i\omega\varepsilon_0 E_0 = 0 & \text{in } \Omega, \\ \nu \wedge E_0|_{\partial\Omega} = \psi \in TH_{\mathrm{Div}}^{-1/2}(\partial\Omega). \end{cases} \tag{1.13}$$

It is assumed that ω is not an EM eigenvalue to the Maxwell equations (1.13), namely, if $\psi = 0$, then one must have $E_0 = H_0 = 0$ for (1.13). Hence we have a well-defined admittance map

$$\Lambda_0(\psi) = \nu \wedge H_0|_{\partial\Omega} : TH_{\mathrm{Div}}^{-1/2}(\partial\Omega) \to TH_{\mathrm{Div}}^{-1/2}(\partial\Omega), \tag{1.14}$$

where $H_0 \in H(\nabla\wedge;\Omega)$ is the unique solution to (1.13). We refer to [33] and [24] for studies on the well-posedness of the Maxwell equations in the function setting introduced above.

We are ready to state the main result of this paper.

THEOREM 1.1. *Suppose ω is not an EM eigenvalue of the free-space Maxwell equations* (1.13). *Let $\widetilde{\Lambda}_\rho$ be the boundary admittance map in* (1.11) *associated with* (1.10), *where the EM parameter tensors are given by* (1.5)–(1.9). *Let Λ_0 be the free admittance map in* (1.14) *associated with* (1.13). *Then there exists a positive constant ρ_0 such that for any $\rho < \rho_0$,*

$$\|\widetilde{\Lambda}_\rho - \Lambda_0\|_{\mathcal{L}(TH_{\mathrm{Div}}^{-1/2}(\partial\Omega), TH_{\mathrm{Div}}^{-1/2}(\partial\Omega))} \le C\rho^3, \tag{1.15}$$

where C is a positive constant dependent only on $\rho_0, \omega, \alpha_0, \beta_0, \gamma_0$ and D, Ω, but completely independent of ρ, $\tilde{\varepsilon}_a$, $\tilde{\mu}_a$ and $\tilde{\sigma}_a$.

Before we proceed to prove the result, some general remarks about the significance of the result are in order.

Theorem 1.1 states that the transformation medium $(\Omega\backslash\overline{D};\varepsilon_c^\rho,\mu_c^\rho,\sigma_c^\rho)$ together with the conducting layer $(D\backslash\overline{D}_{1/2};\varepsilon_l,\mu_l,\sigma_l)$ in (1.9) produces an approximate invisibility cloaking device which nearly cloaks an arbitrary target medium $(D_{1/2};\tilde{\varepsilon}_a,\tilde{\mu}_a,\tilde{\sigma}_a)$ located in the innermost region. Indeed, in the limiting case with $\rho=0$ within spherical geometry, namely, Ω and D are both Euclidean balls, (1.9) without the conducting layer yields the perfect invisibility cloaking construction in [15, 37]. That is, $\Lambda_\rho=\Lambda_0$ for $\rho=0$, and hence by using the exterior boundary measurements encoded in Λ_ρ, one cannot "see" the inside object. However, it is widely known that the construction employs singular materials, that is, the material tensors ε_c^ρ and μ_c^ρ in the limiting case $\rho=0$ possess degenerate singularities (cf. [15]). This presents a great challenge for both theoretical analysis and practical fabrications. In order to avoid the singular structure/materials, several regularized constructions have been developed. In [13, 14, 38], a truncation of singularities has been introduced. In [22, 23, 29], the "blow-up-a-point" transformation in [18, 19, 25, 37] has been regularized to become the "blow-up-a-small-region" transformation. Nevertheless, as pointed out in [21], the truncation-of-singularity construction and the blow-up-a-small-region construction are equivalent to each other. Hence, our present study focuses on the blow-up-a-small-region construction, that is, F_ρ is used to blow up D_ρ of the relative size $\rho\ll 1$ for constructing the cloaking medium in (1.5). Theorem 1.1 states that our cloaking construction (1.9) yields an approximate cloaking device within ρ^3-accuracy of the perfect cloak. Furthermore, an arbitrary content can be nearly cloaked.

Due to its practical importance, the approximate cloaking has recently been extensively studied. In [1, 23], approximate cloaking schemes were developed for electric impedance tomography which might be regarded as optics at zero frequency. In [2, 3, 22, 26, 28, 29], various near-cloaking schemes were presented for scalar waves governed by the Helmholtz equation. In [30], a similar construction to (1.9) was developed for the full Maxwell equations. However, the study in [30] was only conducted for spherical geometry and the uniform cloaked content, that is, both Ω and D in (1.9) were assumed to be Euclidean balls and the medium parameters $\tilde{\varepsilon}_a$, $\tilde{\mu}_a$, and $\tilde{\sigma}_a$ were all constants multiple of the identity matrix. Under these assumptions, the Fourier–Bessel technique can be used to derive the analytic series expansions of the EM fields [30]. To our best knowledge, Theorem 1.1 is the first result for nearly cloaking the full Maxwell equations with general geometry and arbitrary cloaked contents. In order to assess the near-cloaking construction, the study is shown to be reduced to the boundary effect estimate due to an inhomogeneous EM inclusion with an asymptotically small support but an arbitrary content enclosed by a thin conducting layer with an asymptotically high conductivity tensor. The new structures of our problem require novel mathematical arguments. Our mathematical analysis uses the general strategy developed in [28] for nearly cloaking of acoustic waves governed by the scalar Helmholtz equation. However, new techniques and estimates must be developed to deal with the general vector Maxwell equations. We point out that incorporating a damping mechanism by a conducting layer into the near-cloaking construction (1.9) is necessary for achieving successful near-cloak. In fact, it has been shown in [30] that no matter how small the regularization parameter ρ is, there always exist cloak-busting inclusions. Moreover, the result in [30] for the special case within spherical geometry and uniform cloaked contents confirms that our estimate in Theorem 1.1 is sharp. Finally, it is remarked that the estimate in (1.15) is frequency-dependent, and hence it is a narrow-band estimate. Deriving the broad-band estimate for our near-cloaking construction is worthy of further investigation. Also, (1.15) is a boundary estimate which means the near-field measurement is used in assessing the cloaking performance. One may also consider assessing the cloaking performance in terms of the far-field measurement where one should be able to derive estimates in stronger norms instead of the weak operator-norm estimate in (1.15).

For noninvasive EM detections, a related inverse problem is to extract physical information of the interior object, namely, $\tilde{\varepsilon}, \tilde{\mu}$, and $\tilde{\sigma}$ from the knowledge of the exterior EM measurements encoded into the boundary operator $\widetilde{\Lambda}_\rho$. We refer the reader to [35] and [36] and the references therein for results on uniqueness and stability of this important inverse problem.

The rest of the paper is organized as follows. In section 2, we present the proof of Theorem 1.1. Section 3 is devoted to the proof of a key lemma that was needed in the proof of Theorem 1.1.

2. Proof of the main theorem.

2.1. Proof of Theorem 1.1. We first present a lemma with some key ingredients of the transformation optics, the proofs of which are available in [30].

LEMMA 2.1. *Suppose that $E \in H(\nabla\wedge;\Omega)$ and $H \in H(\nabla\wedge;\Omega)$ are EM fileds satisfying*

$$\nabla\wedge E - i\omega\mu H = 0 \qquad in\ \ \Omega,$$
$$\nabla\wedge H + i\omega\left(\varepsilon + i\frac{\sigma}{\omega}\right)E = 0 \qquad in\ \ \Omega,$$

where $(\Omega;\varepsilon,\mu,\sigma)$ is a regular EM medium. Let $x' = \mathcal{F}(x) : \Omega \to \Omega$ be a bi-Lipschitz and orientation-preserving mapping such that $\mathcal{F}|_{\partial\Omega} =$ Identity. Define the pull-back EM fields by

$$E' = (\mathcal{F}^{-1})^* E := (D\mathcal{F})^{-T} E \circ \mathcal{F}^{-1},$$
$$H' = (\mathcal{F}^{-1})^* H := (D\mathcal{F})^{-T} H \circ \mathcal{F}^{-1}.$$

Then, $E' \in H(\nabla'\wedge;\Omega)$ and $H' \in H(\nabla'\wedge;\Omega)$. In addition the following identities hold:

$$\nabla'\wedge E' = i\omega\mu' H' \qquad in\ \ \Omega,$$
$$\nabla'\wedge H' = -i\omega\left(\varepsilon' + i\frac{\sigma'}{\omega}\right)E' \qquad in\ \ \Omega,$$

where $\nabla'\wedge$ denote the curl *operator in the x'-coordinates, and ε', μ', and σ' are the push-forwards of ε, μ, and σ via $\mathcal{F}$, namely,*

$$(\Omega;\varepsilon',\mu',\sigma') = \mathcal{F}_*(\Omega;\varepsilon,\mu,\sigma).$$

Particularly, if one lets Λ and Λ' denote the admittance maps associated with (E,H) and (E',H'), respectively, then

$$\Lambda = \Lambda'.$$

Next, for the EM fields $(\widetilde{E}_\rho, \widetilde{H}_\rho)$ in (1.10) associated with the boundary condition (1.12), we let

$$E_\rho = F^* \widetilde{E}_\rho \quad \text{and} \quad H_\rho = F^* \widetilde{H}_\rho. \tag{2.1}$$

Then by Lemma 2.1, it is seen that $E_\rho \in H(\nabla\wedge;\Omega)$ and $H_\rho \in H(\nabla\wedge;\Omega)$, which satisfy the following Maxwell equations:

$$\begin{cases} \nabla\wedge E_\rho - i\omega\mu_\rho H_\rho = 0 & \text{in } \Omega, \\ \nabla\wedge H_\rho + i\omega\left(\varepsilon_\rho + i\dfrac{\sigma_\rho}{\omega}\right) E_\rho = 0 & \text{in } \Omega, \\ \nu\wedge E_\rho|_{\partial\Omega} = \psi \in TH_{\mathrm{Div}}^{-1/2}(\partial\Omega), \end{cases} \tag{2.2}$$

where

$$\Omega; \varepsilon_\rho, \mu_\rho, \sigma_\rho = \begin{cases} \varepsilon_0, \mu_0, 0 & \text{in } \Omega\backslash D_\rho, \\ \alpha_0\varepsilon_0, \beta_0\rho^2\mu_0, \gamma_0\rho^{-2}\delta & \text{in } D_\rho\backslash\overline{D}_{\rho/2}, \\ \varepsilon_a, \mu_a, \sigma_a & \text{in } D_{\rho/2}. \end{cases} \tag{2.3}$$

Furthermore, by Lemma (2.1),

$$\Lambda_\rho = \widetilde{\Lambda}_\rho, \tag{2.4}$$

where Λ_ρ is the admittance map associated with the EM fileds (E_ρ, H_ρ) in (2.2). Hence, in order to prove Theorem 1.1, it suffices to show the following result.

THEOREM 2.1. *Suppose ω is not an EM eigenvalue of the free-space Maxwell equations* (1.13). *Let $(E_0, H_0) \in H(\nabla\wedge;\Omega)\wedge H(\nabla\wedge;\Omega)$ and $(E_\rho, H_\rho) \in H(\nabla\wedge;\Omega)\wedge H(\nabla\wedge;\Omega)$ be solutions to* (1.13) *and* (2.2)*, respectively. Then there exists a positive constant ρ_0 such that for any $\rho < \rho_0$,*

$$\|\nu\wedge H_\rho - \nu\wedge H_0\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \le C\rho^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}, \tag{2.5}$$

where C is a positive constant dependent only on $\rho_0, \omega, \alpha_0, \beta_0, \gamma_0$ and D, Ω, but completely independent of ρ, ε_a, μ_a and σ_a and ψ.

Remark 1. Qualitatively speaking, Theorem 2.1 states that the boundary EM effects due to an inhomogeneous EM inclusion supported in a small region D_ρ with arbitrary contents in $D_{\rho/2}$ but enclosed by a thin layer of high conducting medium in $D_\rho\backslash\overline{D}_{\rho/2}$ is also small. In the literature, there are extensive studies on the scattering estimates due to small EM scatterers (see [4, 6, 7]), and also the closely related low-frequency asymptotics of EM scattering (i.e., the Rayleigh approximation; see [5, 12, 31, 33]). However, the EM inclusions of the aforementioned studies have fixed contents, and indeed they are either perfectly conducting obstacles or EM mediums with piecewise constant material parameters. In Theorem 2.1, the small EM inclusion of our current study has peculiar structures, and the quantitative estimate (2.5) cannot be adapted from existing results in the literature.

The rest of the paper is devoted to the proof of Theorem 2.1. In order to simplify the exposition, we set

$$\alpha_0 = \beta_0 = \gamma_0 = 1.$$

We next derive three key lemmas.

LEMMA 2.2. *The solutions of* (1.13) *and* (2.2) *satisfy*

$$\int_{D_\rho\backslash D_{\rho/2}} |E_\rho|^2\, d\sigma_x \le C\rho^2\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}, \tag{2.6}$$

where C is a positive constant depending only on Ω.

Proof. Inner-producting both sides of the second equation in (2.2) by $\overline{E}_\rho$, and integrating by parts, we have

$$\begin{aligned}&\int_\Omega -i\omega\left(\varepsilon_\rho+i\frac{\sigma_\rho}{\omega}\right)E_\rho\cdot\overline{E}_\rho\ d\sigma_x=\int_\Omega(\nabla\wedge H_\rho)\cdot\overline{E}_\rho\ d\sigma_x\\ &=\int_\Omega H_\rho\cdot(\nabla\wedge\overline{E}_\rho)\ d\sigma_x-\int_{\partial\Omega}(\nu\wedge\overline{E}_\rho)\cdot H_\rho\ ds_x\\ &=\int_\Omega H_\rho\cdot(-i\omega\overline{\mu_\rho}\overline{H}_\rho)\ d\sigma_x+\int_{\partial\Omega}(\nu\wedge\overline{E}_\rho)\cdot[\nu\wedge(\nu\wedge H_\rho)]\ ds_x.\end{aligned}\tag{2.7}$$

In (2.7), we have made use of the decomposition that

$$H_\rho|_{\partial\Omega}=(H_\rho|_{\partial\Omega})_t+\nu(H_\rho|_{\partial\Omega})_\nu,$$

where the tangential component $(H_\rho|_{\partial\Omega})_t$ is $-\nu\wedge(\nu\wedge(H_\rho|_{\partial\Omega}))$ and the normal component is $(H_\rho|_{\partial\Omega})_\nu$ is $\langle\nu,H_\rho|_{\partial\Omega}\rangle$. By taking the real parts of both sides of (2.7), we have

$$\begin{aligned}&\int_{D_{\rho/2}}\sigma_a E_\rho\cdot\overline{E}_\rho\ d\sigma_x+\rho^{-2}\int_{D_\rho\backslash D_{\rho/2}}|E_\rho|^2\ d\sigma_x\\ &=\Re\int_{\partial\Omega}(\nu\wedge\overline{E}_\rho)\cdot[\nu\wedge(\nu\wedge H_\rho)]\ ds_x.\end{aligned}\tag{2.8}$$

On the other hand, it is straightforward to verify that

$$0=\Re\int_{\partial\Omega}(\nu\wedge\overline{E}_0)\cdot[\nu\wedge(\nu\wedge H_0)]\ ds_x.\tag{2.9}$$

We shall make use of the following fact that the skew-symmetric bilinear form

$$\begin{aligned}\mathcal{B}:TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)\wedge TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)\ &\to\ \ \mathbb{C},\\ (\mathbf{j},\mathbf{m})\qquad\qquad &\to\ \ \mathcal{B}(\mathbf{j},\mathbf{m})=\int_{\partial\Omega}\mathbf{j}\cdot(\mathbf{m}\wedge\nu)\ ds\end{aligned}\tag{2.10}$$

defines a nondegenerate duality product on $TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)$ (cf. [11]). Subtracting (2.9) from (2.8), one has

$$\begin{aligned}&\int_{D_{\rho/2}}\sigma_a E_\rho\cdot\overline{E}_\rho\ d\sigma_x+\rho^{-2}\int_{D_\rho\backslash D_{\rho/2}}|E_\rho|^2\ d\sigma_x\\ &=\Re\int_{\partial\Omega}\overline{\psi}\cdot[\nu\wedge(\nu\wedge(H_\rho-H_0))]\ ds_x,\end{aligned}$$

which together with the duality (2.10) yields (2.6). ∎

In what follows, we let

$$\nu\wedge E^-_\rho(x)\quad(\text{resp., }\ \nu\wedge H^-_\rho(x))\quad\text{on}\quad\partial D_\rho$$

denote the tangential component of E_ρ (resp., H_ρ) on ∂D_ρ when one approaches ∂D_ρ from the interior of D_ρ. Similarly, we let

$$\nu\wedge E^+_\rho(x)\quad(\text{resp., }\ \nu\wedge H^-_\rho(x))\quad\text{on}\quad\partial D_\rho$$

denote the tangential component of E_ρ (resp., H_ρ) on ∂D_ρ when one approaches D_ρ from the exterior of D_ρ.

LEMMA 2.3. *The solutions to* (1.13) *and* (2.2) *satisfy*

$$\begin{aligned} &\left\|(\nu\wedge E_\rho^-)(\rho\ \cdot)\right\|^2_{TH^{-1/2}(\partial D)}\\ &\quad\leq C\rho^{-1}\left|1+\omega^2\rho^2\left(1+i\frac{\rho^{-2}}{\omega}\right)\right|^2\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}\left\|\nu\wedge(H_\rho-H_0)\right\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}, \end{aligned}\tag{2.11}$$

and hence by the transmission condition across ∂D_ρ

$$\begin{aligned} &\left\|(\nu\wedge E_\rho^+)(\rho\ \cdot)\right\|^2_{TH^{-1/2}(\partial D)}\\ &\quad\leq C\rho^{-1}\left|1+\omega^2\rho^2\left(1+i\frac{\rho^{-2}}{\omega}\right)\right|^2\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}\left\|\nu\wedge(H_\rho-H_0)\right\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}, \end{aligned}\tag{2.12}$$

where C is a positive constant dependent only on D and Ω, but independent of ψ and ρ.

Proof. We let $E_\rho=(E_\rho^1,E_\rho^2,E_\rho^3)$. Clearly, it suffices to show that for $k=1,2,3$,

$$\begin{aligned} &\left\|E_\rho^k(\rho\ \cdot)\right\|^2_{H^{-1/2}(\partial D)}\\ &\quad\leq C\rho^{-1}\left|1+\omega^2\rho^2\left(1+i\frac{\rho^{-2}}{\omega}\right)\right|^2\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}\left\|\nu\wedge(H_\rho-H_0)\right\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)}. \end{aligned}\tag{2.13}$$

Our proof begins with the following duality identity:

$$\left\|E_\rho^k(\rho\ \cdot)\right\|_{H^{-1/2}(\partial D)}=\sup_{\|\phi\|_{H^{1/2}(\partial\Omega)}\leq 1}\left|\int_{\partial D}E_\rho^k(\rho x)\cdot\phi(x)\ ds_x\right|.\tag{2.14}$$

For any $\phi\in H^{1/2}(\partial D)$, there exists $u\in H^2(D)$ such that (see Theorem 14.1 in [41])

(i) $u=0$ on ∂D,

(ii) $\frac{\partial u}{\partial\nu}=\phi$ on ∂D,

(iii) $\|u\|_{H^2(D)}\leq C\|\phi\|_{H^{1/2}(\partial D)}$,

(iv) $u=0$ in $D_{1/2}$.

Then

$$\int_{\partial D}E_\rho^k(\rho x)\cdot\phi(x)\ ds_x=\int_{\partial D}E_\rho^k(\rho x)\cdot\frac{\partial u(x)}{\partial\nu(x)}\ ds_x.\tag{2.15}$$

For $y\in D_\rho$, let

$$x:=\frac{y}{\rho}\in D.$$

Set

$$E(x):=E_\rho(\rho x)=E_\rho(y),\quad x\in D.$$

Since

$$\begin{aligned} &\nabla_y \wedge E_\rho(y) - i\omega H_\rho(y) = 0, \\ &\nabla_y \wedge H_\rho(y) + i\omega \left(1 + i\frac{\rho^{-2}}{\omega}\right) E_\rho(y) = 0, \end{aligned} \tag{2.16}$$

or $y \in D_\rho \backslash \overline{D}_{\rho/2}$, it is easily verified that

$$\begin{aligned} &\nabla_x \wedge E(x) = i\omega\rho H(x), \\ &\nabla_x \wedge H(x) = -i\omega\rho \left(1 + i\frac{\rho^{-2}}{\omega}\right) E(x), \quad x \in D\backslash\overline{D}_{1/2}. \end{aligned} \tag{2.17}$$

Then, by (2.15)–(2.17) and Green's formula, we have

$$\begin{aligned} \int_{\partial D} E_\rho^k(\rho x) \cdot \phi(x)\ ds_x &= \int_{\partial D} E^k(x) \cdot \frac{\partial u}{\partial \nu}(x)\ ds_x \\ &= \int_{\partial D} E^k(x) \cdot \frac{\partial u}{\partial \nu}(x) - \frac{\partial E^k}{\partial \nu}(x) \cdot u(x)\ ds_x \\ &= \int_D E^k(x) \cdot \Delta u(x) - \Delta E^k(x) \cdot u(x)\ d\sigma_x. \end{aligned} \tag{2.18}$$

By (2.17), it is straightforward to show that

$$\Delta E(x) + \omega^2\rho^2 \left(1 + i\frac{\rho^{-2}}{\omega}\right) E(x) = 0, \quad x \in D\backslash\overline{D}_{1/2}. \tag{2.19}$$

Therefore, from (2.18) and (2.19), it follows directly that

$$\begin{aligned} &\int_{\partial D} E_\rho^k(\rho x) \cdot \phi(x)\ ds_x \\ &\quad = \int_D E^k \cdot \Delta u - u \cdot \Delta E^k\ d\sigma_x \\ &\quad = \left[1 + \omega^2\rho^2 \left(1 + i\frac{\rho^{-2}}{\omega}\right)\right] \int_{D\backslash D_{1/2}} E^k \cdot (u + \Delta u)\ d\sigma_x, \end{aligned} \tag{2.20}$$

and hence

$$\begin{aligned} &\left| \int_{\partial D} E_\rho^k(\rho x) \cdot \phi(x)\ ds_x \right| \\ &\quad \le \left| 1 + \omega^2\rho^2 \left(1 + i\frac{\rho^{-2}}{\omega}\right) \right| \|E\|_{L^2(D\backslash D_{1/2})} \|\phi\|_{H^{1/2}(\partial D)^3}. \end{aligned} \tag{2.21}$$

Using the relation

$$\|E\|_{L^2(D\backslash D_{1/2})} = \|E_\rho(\rho\ \cdot)\|_{L^2(D\backslash D_{1/2})} = \rho^{-3/2}\|E_\rho\|_{L^2(D_\rho\backslash D_{\rho/2})},$$

we have from (2.21) that

$$\begin{aligned} &\|E_\rho^k(\rho\ \cdot)\|_{H^{-1/2}(\partial D)} \\ &\quad \le \left| 1 + \omega^2\rho^2 \left(1 + i\frac{\rho^{-2}}{\omega}\right) \right| \|E\|_{L^2(D\backslash D_{1/2})} \\ &\quad \le \rho^{-3/2} \left| 1 + \omega^2\rho^2 \left(1 + i\frac{\rho^{-2}}{\omega}\right) \right| \|E_\rho\|_{L^2(D_\rho\backslash D_{\rho/2})}, \end{aligned}$$

which together with (2.6) in Lemma 2.2 immediately implies (2.13).

The proof is now completed. ∎

The next lemma is of crucial importance, and its proof will be given in section 3.

LEMMA 2.4. *Suppose ω is not an eigenvalue of the free-space Maxwell equations* (1.13). *Let $E_0 \in H(\nabla\wedge;\Omega)$ and $H_0 \in H(\nabla\wedge;\Omega)$ be the solutions to* (1.13). *Let $\tau \in \mathbb{R}_+$ and let*

$$\varphi \in TH_{\mathrm{Div}}^{-1/2}(\partial D_\tau).$$

Consider the Maxwell equations

$$\left\{\begin{array}{ll} \nabla\wedge E_\tau - i\omega H_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nabla\wedge H_\tau + i\omega E_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nu\wedge E_\tau = \varphi \quad \text{on } \partial D_\tau, & \\ \nu\wedge E_\tau = \psi \quad \text{on } \partial\Omega. & \end{array}\right. \tag{2.22}$$

Then there exists a constant $\tau_0 \in \mathbb{R}_+$ such that for any $\tau < \tau_0$,

$$\begin{aligned} &\|\nu\wedge(H_\tau - H_0)\|_{TH^{-1/2}(\partial\Omega)} \\ &\quad \le C\left(\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right), \end{aligned} \tag{2.23}$$

where C is a generic positive constant dependent only on τ_0, ω and Ω, D, but independent of τ and φ, ψ.

It should be emphasized that in the estimate (2.23), the norm for $\varphi(\tau\ \cdot)$ is $TH^{-1/2}(\partial D)$ though $\varphi(\tau\ \cdot) \in TH_{\mathrm{Div}}^{-1/2}(\partial D)$, and in this sense, the estimate is "nonstandard."

We are in position to present the proof of Theorem 1.1.

Proof of Theorem 1.1. By taking $\tau = \rho$ and $\varphi = \nu\wedge E_\rho^+|_{\partial D_\rho}$ in Lemma 2.4, we have

$$\begin{aligned} &\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \\ &\quad \le C_1\left(\rho^3\|\psi\|_{TH_{\mathrm{Div}}}^{-1/2}(\partial\Omega) + \rho^2\|(\nu\wedge E_\rho^+)(\rho\ \cdot)\|_{TH^{-1/2}(\partial D)}\right). \end{aligned} \tag{2.24}$$

Next, by Lemma 2.3, we have for $\kappa > 0$

$$\begin{aligned} &\|(\nu\wedge E_\rho^+)(\rho\ \cdot)\|_{TH^{-1/2}(\partial D)} \\ &\quad \le C_2\rho^{-1/2}\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}^{1/2}\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}^{1/2} \\ &\quad \le C_2\rho^{-1/2}\left(\frac{\rho^{3/2}}{4\kappa}\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \frac{\kappa}{\rho^{3/2}}\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}\right). \end{aligned} \tag{2.25}$$

From (2.24) and (2.25), we further have

$$\begin{aligned} \|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \le{}& C_1\rho^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \frac{1}{4}C_1C_2\frac{\rho^3}{\kappa}\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \\ &+ C_1C_2\kappa\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}. \end{aligned} \tag{2.26}$$

By choosing κ such that $C_1C_2\kappa < 1/2$, we see immediately from (2.26) that

$$\|\nu\wedge(H_\rho - H_0)\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \le C\rho^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)},$$

which completes the proof. ∎

3. Proof of Lemma 2.4. We present the proof of Lemma 2.4, which is crucial for the proof of our main theorem.

Set

$$\widetilde{E}_\tau = E_\tau - E_0, \qquad \widetilde{H}_\tau = H_\tau - H_0.$$

It is straightforward to verify that

$$\left\{\begin{array}{ll} \nabla\wedge\widetilde{E}_\tau - i\omega\widetilde{H}_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nabla\wedge\widetilde{H}_\tau + i\omega\widetilde{E}_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nu\wedge\widetilde{E}_\tau = \varphi - \nu\wedge E_0 & \text{on } \partial D_\tau, \\ \nu\wedge\widetilde{E}_\tau = 0 & \text{on } \partial\Omega. \end{array}\right. \tag{3.1}$$

Obviously, in order to show (2.23), it suffices to show

$$\|\nu\wedge\widetilde{H}_\tau\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} \le C\left(\tau^3\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right). \tag{3.2}$$

In order to prove (3.2), we let

$$\widetilde{E}_\tau = U_\tau - \widetilde{U}_\tau \qquad \text{and} \qquad \widetilde{H}_\tau = V_\tau - \widetilde{V}_\tau, \tag{3.3}$$

where $(U_\tau, V_\tau) \in H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau) \wedge H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau)$ are scattering solutions to

$$\begin{array}{ll} \nabla\wedge U_\tau - i\omega V_\tau = 0 & \text{in } \mathbb{R}^3\backslash\overline{D}_\tau, \\ \nabla\wedge V_\tau + i\omega U_\tau = 0 & \text{in } \mathbb{R}^3\backslash\overline{D}_\tau, \\ \nu\wedge U_\tau = \varphi - \nu\wedge E_0 & \text{on } \partial D_\tau, \\ \lim_{|x|\to+\infty}|x|\left|(\nabla\wedge U_\tau)(x)\wedge\dfrac{x}{|x|} - i\omega U_\tau(x)\right| = 0, & \end{array} \tag{3.4}$$

and $(\widetilde{U}_\tau, \widetilde{V}_\tau) \in H(\nabla\wedge; \Omega\backslash\overline{D}_\tau) \wedge H(\nabla\wedge; \Omega\backslash\overline{D}_\tau)$ are solutions to

$$\begin{array}{ll} \nabla\wedge\widetilde{U}_\tau - i\omega\widetilde{V}_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nabla\wedge\widetilde{V}_\tau + i\omega\widetilde{U}_\tau = 0 & \text{in } \Omega\backslash\overline{D}_\tau, \\ \nu\wedge\widetilde{U}_\tau = \nu\wedge U_\tau & \text{on } \partial\Omega, \\ \nu\wedge\widetilde{U}_\tau = 0 & \text{on } \partial D_\tau. \end{array} \tag{3.5}$$

We shall show the following two lemmas, which immediately imply (3.2).

Lemma 3.1. *Let $(U_\tau, V_\tau) \in H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau) \wedge H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau)$ be scattering solutions to* (3.4)*. Then there exists $\tau_0 \in \mathbb{R}_+$ such that for any $\tau < \tau_0$*

$$\|U_\tau\|_{(C^2(\partial\Omega))^3} \le C\left(\tau^3\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right) \tag{3.6}$$

and

$$\|V_\tau\|_{(C^2(\partial\Omega))^3} \le C\left(\tau^3\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right), \tag{3.7}$$

where C is a positive constant dependent only on τ_0, ω and Ω, D, but independent of τ and φ, ψ.

Lemma 3.2. *Let $(\widetilde{U}_\tau, \widetilde{V}_\tau) \in H(\nabla\wedge; \Omega\backslash\overline{D}_\tau) \wedge H(\nabla\wedge; \Omega\backslash\overline{D}_\tau)$ be solutions to* (3.5)*. Then there exists $\tau_0 \in \mathbb{R}_+$ such that for any $\tau < \tau_0$*

$$\|\nu\wedge\widetilde{V}_\tau\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} \le C\left(\tau^3\|\psi\|_{TH^{-1/2}_{\mathrm{Div}}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right), \tag{3.8}$$

where C is a positive constant dependent only on τ_0, ω and Ω, D, but independent of τ and φ, ψ.

We next present the proof of Lemma 3.1, which may be further divided into the following two propositions.

PROPOSITION 3.1. *Let $(U_{\tau,1}, V_{\tau,1}) \in H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau) \wedge H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau)$ be solutions to*

$$\begin{aligned} \nabla\wedge U_{\tau,1} - i\omega V_{\tau,1} &= 0 \qquad \text{in } \mathbb{R}^3\backslash\overline{D}_\tau,\\ \nabla\wedge V_{\tau,1} + i\omega U_{\tau,1} &= 0 \qquad \text{in } \mathbb{R}^3\backslash\overline{D}_\tau,\\ \nu\wedge U_{\tau,1} &= \nu\wedge E_0 \quad \text{on } \partial D_\tau,\\ \lim_{|x|\to+\infty} |x| \left| (\nabla\wedge U_{\tau,1})(x)\wedge\frac{x}{|x|} - i\omega U_{\tau,1}(x)\right| &= 0. \end{aligned} \tag{3.9}$$

Then there exists $\tau_0 \in \mathbb{R}_+$ such that for any $\tau < \tau_0$

$$\|U_{\tau,1}\|_{(C^2(\partial\Omega))^3} \le C\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \tag{3.10}$$

and

$$\|V_{\tau,1}\|_{(C^2(\partial\Omega))^3} \le C\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}, \tag{3.11}$$

where C is a positive constant dependent only on τ_0, ω and Ω, D, but independent of τ and φ, ψ.

PROPOSITION 3.2. *Let $(U_{\tau,2}, V_{\tau,2}) \in H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau) \wedge H_{loc}(\nabla\wedge; \mathbb{R}^3\backslash\overline{D}_\tau)$ be solutions to*

$$\begin{aligned} \nabla\wedge U_{\tau,2} - i\omega V_{\tau,2} &= 0 \quad \text{in } \mathbb{R}^3\backslash\overline{D}_\tau,\\ \nabla\wedge V_{\tau,2} + i\omega U_{\tau,2} &= 0 \quad \text{in } \mathbb{R}^3\backslash\overline{D}_\tau,\\ \nu\wedge U_{\tau,2} &= \varphi \quad \text{on } \partial D_\tau,\\ \lim_{|x|\to+\infty} |x| \left| (\nabla\wedge U_{\tau,2})(x)\wedge\frac{x}{|x|} - i\omega U_{\tau,2}(x)\right| &= 0. \end{aligned} \tag{3.12}$$

Then there exists $\tau_0 \in \mathbb{R}_+$ such that for any $\tau < \tau_0$

$$\|U_{\tau,2}\|_{(C^2(\partial\Omega))^3} \le C\tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)} \tag{3.13}$$

and

$$\|V_{\tau,2}\|_{(C^2(\partial\Omega))^3} \le C\tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}, \tag{3.14}$$

where C is a positive constant dependent only on τ_0, ω and Ω, D, but independent of τ and φ, ψ.

Proof of Proposition 3.1. We first note that the solutions E_0 and H_0 to (1.13) are smooth inside Ω, and also by the local regularity estimate we have

$$\|E_0\|_{(C^1(\overline{D}))^3} \le C\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} \text{ and } \|H_0\|_{(C^1(\overline{D}))^3} \le C\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)}, \tag{3.15}$$

where C is constant depending only on Ω, D and ω. Since $\nu\wedge E_0$ is smooth on ∂D_τ, we know both $U_{\tau,1}$ and $V_{\tau,1}$ are smooth solutions to (3.9) (see [9, 10]). Introducing the scaled fields

$$\mathbf{u}_{\tau,1}(x) = U_{\tau,1}(\tau x), \quad \mathbf{v}_{\tau,1} = V_{\tau,1}(\tau x), \quad x\in\mathbb{R}^3\backslash\overline{D},$$

one can easily verify that

$$\begin{aligned}
\nabla\wedge\mathbf{u}_{\tau,1}-i(\omega\tau)\mathbf{v}_{\tau,1}&=0 \qquad \text{in } \mathbb{R}^3\backslash\overline{D},\\
\nabla\wedge\mathbf{v}_{\tau,1}+i(\omega\tau)\mathbf{u}_{\tau,1}&=0 \qquad \text{in } \mathbb{R}^3\backslash\overline{D},\\
\nu\wedge\mathbf{u}_{\tau,1}&=\nu\wedge\mathbf{e}_\tau \quad \text{on } \partial D,\\
\lim_{|x|\to+\infty}|x|\left|(\nabla\wedge\mathbf{u}_{\tau,1})(x)\wedge\frac{x}{|x|}-i\omega\mathbf{u}_{\tau,1}(x)\right|&=0,
\end{aligned}\tag{3.16}$$

where $\mathbf{e}_\tau(x)=E_0(\tau x)$ for $x\in\partial D$. When $\tau\to+0$, (3.16) is a low frequency EM scattering problem (as $\omega\tau\to+0$) with an incident wave that behaves like the constant field $(E_0(0),H_0(0))$. The asymptotic behavior of the low frequency scattering problem is now well known (cf. [12]), from which one can readily deduce the desired estimates in (3.10) and (3.11). It is remarked that the above argument is similar to the one used in deriving Corollary 3.2 in [7].

The proof is completed. □

Proof of Proposition 3.2. We make use of the layer potential technique to show the lemma. To that end, we let

$$G(x,y):=\frac{1}{4\pi}\frac{e^{i\omega|x-y|}}{|x-y|}\quad\text{and}\quad G_0(x,y):=\frac{1}{4\pi}\frac{1}{|x-y|},\ x,y\in\mathbb{R}^3;\ x\neq y.$$

Furthermore, we introduce the following vector boundary layer potential operators $\mathbf{M}_\Gamma$ and $\mathbf{M}_\Gamma^0$:

$$(\mathbf{M}_\Gamma\mathbf{a})(x):=2\int_\Gamma\nu(x)\wedge[\nabla_x\wedge(\mathbf{a}(y)G(x,y)]\ ds_y,\quad x\in\Gamma\tag{3.17}$$

and

$$(\mathbf{M}_\Gamma^0\mathbf{a})(x):=2\int_\Gamma\nu(x)\wedge[\nabla_x\wedge(\mathbf{a}(y)G_0(x,y)]\ ds_y,\quad x\in\Gamma,\tag{3.18}$$

where $\mathbf{a}$ is a tangential vector field on Γ. We refer to [9, 32, 33, 39] for related mapping properties of the above introduced operators.

We make use of the following *ansatz* of the EM fields to (3.12):

$$U_{\tau,2}(x)=\nabla_x\wedge\int_{\partial D_\tau}G(x,y)\mathbf{a}(y)\ ds_y,\quad x\in\mathbb{R}^3\backslash\overline{D}_\tau,\tag{3.19}$$

$$\begin{aligned}
V_{\tau,2}(x)=\frac{1}{i\omega}\nabla_x\wedge U_{\tau,2}(x)&=\frac{\omega}{i}\int_{\partial D_\tau}G(x,y)\mathbf{a}(y)\ ds_y\\
&\quad+\frac{1}{i\omega}\int_{\partial D_\tau}\nabla_x G(x,y)\operatorname{Div}\mathbf{a}(y)\ ds_y,\quad x\in\mathbb{R}^3\backslash\overline{D}_\tau,
\end{aligned}\tag{3.20}$$

where $\mathbf{a}\in TH_{\mathrm{Div}}^{-1/2}(\partial D_\tau)$. By letting x approach ∂D_τ^+ and using the mapping properties of $\mathbf{M}_{\partial D_\tau}$ and the jump properties of the vector potential operator $\mathbf{M}_{\partial D_\tau}$, one has

$$\mathbf{a}(x)+[\mathbf{M}_{\partial D_\tau}\mathbf{a}](x)=2\varphi(x),\quad x\in\partial D_\tau.\tag{3.21}$$

We claim that there exists $\tau_0\in\mathbb{R}_+$ sufficiently small such that when $\tau<\tau_0$, there exists a unique $\mathbf{a}\in TH_{\mathrm{Div}}^{-1/2}(\partial D_\tau)$ to (3.21), and, moreover,

$$\|\mathbf{a}(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\leq C\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)},\tag{3.22}$$

where C is a generic constant independent of τ and φ. To that end, we let

$$\widetilde{\mathbf{a}}(x') =: \mathbf{a}(\tau x'), \quad x' := \frac{x}{\tau} \in \partial D, \quad x \in \partial D_\tau.$$

By using change of variables,

$$x' = x/\tau \text{ and } y' = y/\tau \text{ for } x, y \in \partial D_\tau, \tag{3.23}$$

one can show

$$\begin{aligned}(\mathbf{M}_{\partial D_\tau}\mathbf{a})(x) &= (\mathbf{M}_{\partial D_\tau}\mathbf{a})(\tau x')\\ &= 2\nu(x') \wedge \nabla_{x'} \wedge \int_{\partial D} G_\tau(x', y')\widetilde{\mathbf{a}}(y')\, ds_{y'},\end{aligned} \tag{3.24}$$

where

$$G_\tau(x', y') = \frac{1}{4\pi}\frac{e^{i\tau\omega|x'-y'|}}{|x'-y'|} = G_0(x', y') + \frac{i\tau\omega}{4\pi} + \tau^2\mathscr{R}(x', y').$$

It is easily verified that the remainder term $\mathscr{R}(x', y')$ satisfies

$$|\mathscr{R}(x', y')| = \mathcal{O}(|x'-y'|), \quad x', y' \in \partial D. \tag{3.25}$$

Hence, we have the following splitting:

$$\begin{aligned}&2\nu(x') \wedge \nabla_{x'} \wedge \int_{\partial D} G_\tau(x', y')\widetilde{\mathbf{a}}(y')\, ds_{y'}\\ &\quad= 2\nu(x') \wedge \nabla_{x'} \wedge \int_{\partial D} G_0(x', y')\widetilde{\mathbf{a}}(y')\, ds_y'\\ &\quad\quad+ 2\tau^2\nu(x') \wedge \nabla_{x'} \wedge \int_{\partial D} \mathscr{R}(x', y')\widetilde{\mathbf{a}}(y')\, ds_{y'}\\ &\quad= (\mathbf{M}^0_{\partial D}\widetilde{\mathbf{a}})(x') + (\mathcal{R}\widetilde{\mathbf{a}})(x').\end{aligned} \tag{3.26}$$

By using the mapping properties of layer potential operators in [33], it is straightforward to show that

$$\begin{aligned}\|\mathcal{R}\widetilde{\mathbf{a}}(\cdot)\|_{TH^{-1/2}(\partial D)} &\le C\tau^2\|\widetilde{\mathbf{a}}(\cdot)\|_{TH^{-1/2}(\partial D)}\\ &= C\tau^2\|\mathbf{a}(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)},\end{aligned} \tag{3.27}$$

where C is a generic constant depending only on D and ω. Then, using again the change of variables in (3.23) to the integral equation (3.21), and the splitting (3.26), one has by direct calculations that

$$[I + M^0_{\partial D} + \mathcal{R}]\widetilde{\mathbf{a}}(x') = 2\varphi(\tau x'), \quad x' \in \partial D. \tag{3.28}$$

Next, we shall show

$$I + M^0_{\partial D} \text{ is invertible from } TH^{-1/2}(\partial D) \text{ to } TH^{-1/2}(\partial D), \tag{3.29}$$

which together with (3.25) and (3.28) implies that

$$\widetilde{\mathbf{a}}(x') = 2[(I + M^0_{\partial D})^{-1} + \mathcal{O}(\tau^2)](\varphi(\tau\ \cdot))(x'), \tag{3.30}$$

thus proving the claim in (3.22). In order to show (3.29), we first note that $M^0_{\partial D}$ is an integral operator of order -1, i.e., it is continuous from $TH^s(\partial D)$ to $TH^{s+1}(\partial D)$ (see [33, p. 242]). Hence, by the Reisz–Fredholm theory, it is sufficient to show that

$$(I + M^0_{\partial D})\mathbf{b} = 0, \quad \mathbf{b} \in TH^{-1/2}(\partial D) \tag{3.31}$$

has only trivial solution, i.e., $\mathbf{b} = 0$. By using the fact that $M^0_{\partial D}$ is of degree -1, we see that the spectrum of $M^0_{\partial D}$ is the same in $TH^{-1/2}(\partial D)$ and $C(\partial D)$. Then by Theorem 5.4 in [9], one must have $\mathbf{b} = 0$ in (3.31), which readily proves (3.29).

Finally, using (3.22) and the integral representations (3.19) and (3.20), direct calculations show (3.13) and (3.14).

The proof is completed. □

Proof of Lemma 3.2. Since U_τ is smooth on $\partial\Omega$, we know both $\widetilde{U}_\tau$ and $\widetilde{V}_\tau$ are strong solutions which belong to $(C^1(\Omega\backslash\overline{D}_\tau) \cap C^{0,\alpha}(\overline{\Omega}\backslash D_\tau))^3$ with $0 < \alpha < 1$ (see [9, 10]). Hence, in what follows, we shall work within the classic setting. To that end, we introduce $T(\Gamma)$, the spaces of all continuous tangential fields $\mathbf{a}$ equipped with the supremum norm, and $T^{0,\alpha}(\Gamma)$, the space of all Hölder continuous tangential fields equipped with the usual Hölder norm. We also need to introduce normed spaces of tangential fields possessing a surface divergence by

$$T_d(\Gamma) := \{\mathbf{a} \in T(\Gamma) | \operatorname{Div} \mathbf{a} \in C(\Gamma)\}$$

and

$$T^{0,\alpha}_d(\Gamma) := \{\mathbf{a} \in T^{0,\alpha} | \operatorname{Div} \mathbf{a} \in C^{0,\alpha}(\Gamma)\}$$

equipped with the norms

$$\|\mathbf{a}\|_{T_d} := \|\mathbf{a}\|_\infty + \|\operatorname{Div} \mathbf{a}\|_\infty, \quad \|a\|_{T^{0,\alpha}_d} := \|a\|_{0,\alpha} + \|\operatorname{Div} \mathbf{a}\|_{0,\alpha}.$$

We again employ the boundary layer potential operators introduced in (3.17) and (3.18), and refer to [9] and [10] for mapping and jumping properties in the classical setting.

Similar to the proof of Proposition 3.2, instead of proving the uniform well-posedness of the Maxwell equations (3.5), we focus on deriving the desired estimate (3.8). However, it should be pointed out that the unique existence of strong solutions to (3.5) for sufficiently small τ can be directly seen from our subsequent argument.

By the Stratton–Chu formula, we have (see [10, Theorem 6.2])

$$\begin{aligned} \widetilde{V}_\tau(x) = &-\nabla_x \wedge \int_{\partial\Omega} \nu(y) \wedge \widetilde{V}_\tau(y) G(x,y)\, ds_y \\ &+ \nabla_x \wedge \int_{\partial D_\tau} \nu(y) \wedge \widetilde{V}_\tau(y) G(x,y)\, ds_y \\ &- \frac{1}{i\omega} \nabla_x \wedge \nabla_x \wedge \int_{\partial\Omega} \nu(y) \wedge U_\tau(y) G(x,y)\, ds_y, \qquad x \in \Omega\backslash\overline{D}_\tau. \end{aligned} \tag{3.32}$$

Set

$$\begin{aligned} \mathbf{a}_1(x) &= \nu(x) \wedge \widetilde{V}_\tau(x), \quad x \in \partial\Omega, \\ \mathbf{a}_2(x) &= \nu(x) \wedge \widetilde{V}_\tau(x), \quad x \in \partial D_\tau, \end{aligned}$$

and

$$P_1(x) = -\frac{1}{i\omega}\nu(x) \wedge \nabla_x \wedge \nabla_x \wedge \int_{\partial\Omega} \nu(y) \wedge U_\tau(y) G(x,y)\, ds_y, \qquad x \in \partial\Omega,$$
$$P_2(x) = -\frac{1}{i\omega}\nu(x) \wedge \nabla_x \wedge \nabla_x \wedge \int_{\partial\Omega} \nu(y) \wedge U_\tau(y) G(x,y)\, ds_y, \qquad x \in \partial D_\tau.$$

By letting x approach ∂D_τ^+, and using the mapping properties of $\mathbf{M}_{\partial D_\tau}$ and $\mathbf{M}_{\partial\Omega}$, one has

$$\begin{aligned} &(I + \mathbf{M}_{\partial\Omega})\mathbf{a}_1(x) - (\mathbf{M}_{\partial D_\tau}\mathbf{a}_2)(x) = 2P_1(x), \quad x \in \partial\Omega,\\ &(I - \mathbf{M}_{\partial D_\tau})\mathbf{a}_2(x) + (\mathbf{M}_{\partial\Omega}\mathbf{a}_1)(x) = 2P_2(x), \quad x \in \partial D_\tau. \end{aligned} \tag{3.33}$$

By using a similar scaling argument to that in the proof of Proposition 3.2, one can show

$$(\mathbf{M}_{\partial D_\tau}\mathbf{a}_2)(\tau x') = (\mathbf{M}^0_{\partial D}\widetilde{\mathbf{a}}_2)(x') + (\mathcal{R}\widetilde{\mathbf{a}}_2)(x'), \quad x' \in \partial D, \tag{3.34}$$

where $\widetilde{\mathbf{a}}_2(x') := \mathbf{a}_2(\tau x')$ for $x' \in \partial D$, and

$$\|\mathcal{R}\widetilde{\mathbf{a}}_2\|_{T_d^{0,\alpha}(\partial D)} \le C\tau^2 \|\widetilde{\mathbf{a}}_2\|_{T_d^{0,\alpha}(\partial D)}. \tag{3.35}$$

By setting $\widetilde{P}_2(x') = P_2(\tau x')$ for $x' \in \partial D$, the system of integral equations (3.33) can be further formulated as

$$\begin{aligned} (I + \mathbf{M}_{\partial\Omega})\mathbf{a}_1(x) - (\widetilde{\mathbf{M}}_{\partial D}\widetilde{\mathbf{a}}_2)(x) &= 2P_1(x), \quad x \in \partial\Omega,\\ (I - M^0_{\partial D})\widetilde{\mathbf{a}}_2(x') - (\mathcal{R}\widetilde{\mathbf{a}}_2)(x') + (\mathbf{M}_{\partial\Omega}\mathbf{a}_1)(\tau x') &= 2\widetilde{P}_2(x'), \quad x' \in \partial D, \end{aligned} \tag{3.36}$$

where

$$(\widetilde{\mathbf{M}}_{\partial D}\widetilde{\mathbf{a}}_2)(x) = (\mathbf{M}_{\partial D_\tau}\mathbf{a}_2)(x), \quad x \in \partial\Omega,$$

obtained by replacing the variable y in defining $\mathbf{M}_{\partial D_\tau}\mathbf{a}_2$ (cf. (3.17)) by $\tau y'$ with $y' \in \partial D$. Since the integral kernel for $(\widetilde{\mathbf{M}}_{\partial D}\widetilde{\mathbf{a}}_2)(x)$ is smooth when $x \in \partial\Omega$, it is straightforwardly shown that

$$\|\widetilde{\mathbf{M}}_{\partial D}\widetilde{\mathbf{a}}_2\|_{T_d^{0,\alpha}(\partial\Omega)} \le C\tau^2 \|\widetilde{\mathbf{a}}_2\|_{T_d^{0,\alpha}(\partial D)}. \tag{3.37}$$

Similarly, one can see that

$$\|\mathbf{M}_{\partial\Omega}\mathbf{a}_1(\tau\ \cdot)\|_{T_d^{0,\alpha}(\partial D)} \le C\|\mathbf{a}_1\|_{T_d^{0,\alpha}(\partial\Omega)}. \tag{3.38}$$

Define

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_1 \\ \widetilde{\mathbf{a}}_2 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 2P_1 \\ 2\widetilde{P}_2 \end{pmatrix} \tag{3.39}$$

and

$$\mathbf{L} = \begin{pmatrix} \mathbf{L}_{11} & \mathbf{0} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} \mathbf{0} & \mathbf{R}_{12} \\ \mathbf{0} & \mathbf{R}_{22} \end{pmatrix}, \tag{3.40}$$

where

$$\mathbf{L}_{11} := I + \mathbf{M}_{\partial\Omega}, \quad \mathbf{L}_{21} := \mathbf{M}_{\partial\Omega}, \quad \mathbf{L}_{22} := I - M^0_{\partial D}$$

and

$$\mathbf{R}_{12} := \widetilde{\mathbf{M}}, \quad \mathbf{R}_{22} := \mathcal{R}.$$

Then the system of integral equations (3.36) can be written as

$$(\mathbf{L} - \mathbf{R})\mathbf{a} = \mathbf{P}. \tag{3.41}$$

$\mathbf{L}_{11}$ is compact from $T_d^{0,\alpha}(\partial\Omega)$ to $T_d^{0,\alpha}(\partial D)$ and $\mathbf{L}_{22}$ is compact from $T_d^{0,\alpha}(\partial D)$ to $T_d^{0,\alpha}(\partial D)$ (cf. [10, Chapter 6]). Since ω is not an EM eigenvalue to (1.13), by the Riesz–Fredholm theory, $\mathbf{L}_{11}$ is invertible (cf. [9, Theorem 4.23]), and $\mathbf{L}_{22}$ is also invertible (cf. [9, Theorem 5.4]). Hence, it is straightforward to verify that $\mathbf{L}$ is invertible from $T_d^{0,\alpha}(\partial\Omega) \wedge T_d^{0,\alpha}(\partial D)$ to itself, and

$$\mathbf{L}^{-1} = \begin{pmatrix} \mathbf{L}_{11}^{-1} & \mathbf{0} \\ -\mathbf{L}_{22}^{-1}\mathbf{L}_{21}\mathbf{L}_{11}^{-1} & \mathbf{L}_{22}^{-1} \end{pmatrix}.$$

Hence, by noting (3.35) and (3.37), we see that for sufficiently small τ,

$$\mathbf{a} = (\mathbf{L} - \mathbf{R})^{-1}\mathbf{P}. \tag{3.42}$$

Finally, by using the mapping property of the electric dipole operator (see Theorem 6.17 in [10]), one can show

$$\|P_1\|_{T_d^{0,\alpha}(\partial\Omega)} \le C\|U_\tau\|_{(C^2(\partial\Omega))^3},$$

which together with Proposition 3.2 further implies that

$$\|P_1\|_{T_d^{0,\alpha}(\partial\Omega)} \le C\left(\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right). \tag{3.43}$$

Moreover, it is straightforward to verify that

$$\|\widetilde{P}_2\|_{T_d^{0,\alpha}(\partial D)} \le C\|U_\tau\|_{(C^2(\partial\Omega))^3}$$

and hence

$$\|\widetilde{P}_2\|_{T_d^{0,\alpha}(\partial D)} \le C\left(\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right). \tag{3.44}$$

By combining (3.42), (3.43), and (3.44), one readily has

$$\|\mathbf{a}_1\|_{T_d^{0,\alpha}(\partial\Omega)} \le C\left(\tau^3\|\psi\|_{TH_{\mathrm{Div}}^{-1/2}(\partial\Omega)} + \tau^2\|\varphi(\tau\ \cdot)\|_{TH^{-1/2}(\partial D)}\right),$$

which immediately implies (3.8).

The proof of Lemma 3.2 is complete. ∎

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REFERENCES

[1] H. Ammari, H. Kang, H. Lee, and M. Lim, *Enhancement of near-cloaking using generalized polarization tensors vanishing structures. Part* I: *The conductivity problem*, Comm. Math. Phys., 317 (2013), pp. 253–266.
[2] H. Ammari, H. Kang, H. Lee, and M. Lim, *Enhancement of near cloaking. Part* II: *The Helmholtz equation*, Comm. Math. Phys., 317 (2013), pp. 485–502.
[3] H. Ammari, J. Garnier, V. Jugnon, H. Kang, M. Lim, and H. Lee, *Enhancement of near-cloaking. Part* III: *Numerical simulations, statistical stability, and related questions*, Contemp. Math., 577 (2012), pp. 1–24.
[4] H. Ammari and H. Kang, *Reconstruction of Small Inhomogeneities from Boundary Measurements*, Lecture Notes in Math. 1846. Springer-Verlag, Berlin, 2004.
[5] H. Ammari and J.-C. Nédélec, *Low-frequency electromagnetic scattering*, SIAM J. Math. Anal., 31 (2000), pp. 836–861.
[6] H. Ammari, M. Vogelius, and D. Volkov, *Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of inhomogeneities of small diameter.* II. *The full Maxwell equations*, J. Math. Pures Appl. (9) (2001), pp. 769–814.
[7] X. Antoine, B. Pincon, K. Ramdani, and B. Thierry, *Far field modeling of electromagnetic time reversal and application to selective focusing on small scatterers*, SIAM J. Appl. Math., 69 (2008), pp. 830–844.
[8] H. Chen and C. T. Chan, *Acoustic cloaking and transformation acoustics*, J. Phys. D: Appl. Phys., 43 (2010), 113001.
[9] D. Colton and R. Kress, *Integral Equations Methods in Scattering Theory*, John Wiley & Sons, New York, 1983.
[10] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, 2nd ed., Springer-Verlag, Berlin, 1998.
[11] M. Costabel and F. L. Louër, *On the Kleinman–Martin integral equation method for electromagnetic scattering by a dielectric body*, SIAM J. Appl. Math., 71 (2011), pp. 635–656.
[12] G. Dassios and R. Kleinman, *Low Frequency Scattering*, Oxford University Press, Oxford, 2000.
[13] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Improvement of cylindrical cloaking with SHS lining*, Optics Express, 15 (2007), pp. 12717–12734.
[14] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Isotropic transformation optics: Approximate acoustic and quantum cloaking*, New J. Phys., 10 (2008), 115024.
[15] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Full-wave invisibility of active devices at all frequencies*, Comm. Math. Phys., 279 (2007), pp. 749–789.
[16] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Invisibility and inverse prolems*, Bull. Amer. Math. Soc., 46 (2009), pp. 55–97.
[17] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, *Cloaking devices, electromagnetic wormholes and transformation optics*, SIAM Rev., 51 (2009), pp. 3–33.
[18] A. Greenleaf, M. Lassas, and G. Uhlmann, *Anisotropic conductivities that cannot detected by EIT*, Physiolog. Meas., 24 (2003).
[19] A. Greenleaf, M. Lassas, and G. Uhlmann, *On nonuniqueness for Calderón's inverse problem*, Math. Res. Lett., 10 (2003), pp. 685–693.
[20] P. Grisvard, *Elliptic Problems in Nonsmooth Domains*, Monogr. Stud. Math. 24, Pitman, Boston, 1985.
[21] I. Kocyigit, H. Y. Liu, and H. Sun, *Regular scattering patterns from near-cloaking devices and their implications for invisibility cloaking*, Inverse Problems, 29 (2013), 045005.
[22] R. Kohn, D. Onofrei, M. Vogelius, and M. Weinstein, *Cloaking via change of variables for the Helmholtz equation*, Comm. Pure Appl. Math., 63 (2010), pp. 973–1016.
[23] R. Kohn, H. Shen, M. Vogelius, and M. Weinstein, *Cloaking via change of variables in electrical impedance tomography*, Inverse Problems, 24 (2008), 015016.
[24] R. Leis, *Initial Boundary Value Problems in Mathematical Physics*, Wiley, Chichester, 1986.
[25] U. Leonhardt, *Optical conformal mapping*, Science, 312 (2006), pp. 1777–1780.
[26] J. Li, H. Y. Liu, and H. Sun, *Enhanced approximate cloaking by SH and FSH lining*, Inverse Problems, 28 (2012), 075011.
[27] J. L. Lions and E. Magenes, *Non-Homogeneous Boundary Value Problems and Applications* I, Springer-Verlag, Berlin, 1970.
[28] H. Y. Liu and H. Sun, *Enhanced near-cloak by FSH lining*, J. Math. Pures Appl., 99 (2013), pp. 17–42.
[29] H. Y. Liu, *Virtual reshaping and invisibility in obstacle scattering*, Inverse Problems, 25 (2009), 045006.

[30] H. Y. Liu and T. Zhou, *On approximate electromagnetic cloaking by transformation media*, SIAM J. Appl. Math., 71 (2011), pp. 218–241.
[31] P. A. Martin, *Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles*, Cambridge University Press, Cambridge, 2006.
[32] W. McLean, *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, 2000.
[33] J. C. Nédélec, *Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems*, Springer-Verlag, Berlin, 2001.
[34] A. N. Norris, *Acoustic cloaking theory*, Proc. Roy. Soc. A, 464 (2008), 2411-434.
[35] P. Ola, L. Päivärinta, and E. Somersalo, *An inverse boundary value problem in electrodynamics*, Duke Math. J., 70 (1993), pp. 617–653.
[36] P. Ola and E. Somersalo, *Electromagnetic inverse problems and generalized Sommerfeld potentials*, SIAM J. Appl. Math., 56 (1996), pp. 1129–1145.
[37] J. B. Pendry, D. Schurig, and D. R. Smith, *Controlling electromagnetic fields*, Science, 312 (2006), pp. 1780–1782.
[38] Z. Ruan, M. Yan, C. W. Neff, and M. Qiu, *Ideal cylyindrical cloak: Perfect but sensitive to tiny perturbations*, Phy. Rev. Lett., 99 (2007), 113903.
[39] M. E. Taylor, *Partial Differential Equations* II: *Qualitative Studies of Linear Equations*, 2nd ed., Springer-Verlag, Berlin, 2011.
[40] G. Uhlmann, *Visibility and invisibility*, Proceedings of ICIAM 07–6th International Congress on Industrial and Applied Mathematics, Eur. Math. Soc., Zürich, 2009, pp. 381–408.
[41] J. Wloka, *Partial Differential Equations*, Cambridge University Press, Cambridge, 1987.
[42] M. Yan, W. Yan, and M. Qiu, *Invisibility cloaking by coordinate transformation*, in Progress in Optics 52, Elsevier, New York, 2008, pp. 261–304.