DOCTORAL THESIS

Mathematical Study on Several Inverse Problems and Invisibility Cloaking with Applications

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Abstract

The study on inverse problems has played a pivotal role to various disciplines of science, technology, engineering and mathematics, including x-ray, ultrasound, magnetoencephalography, geophysical exploration, radar and criminal investigations. In the view of their novel promising applications, we investigate the potentials for several inverse scattering approaches and applications.

In our first topic, we are concerned a new approach for generating two-dimensional or three-dimensional geometric body shapes by inputting characteristic parameters of a specific geometric body. Our study is combined with the machine learning approach and the inverse scattering techniques on the theory of wave propagation associated with the Helmholtz equation. We first introduce the important notations of the shape space and then the shape generators via inverse source scattering associated with Helmholtz equation for the generation of the geometric body shapes. Then, we develop a machine learning scheme for the generation of geometric body shape by using the setup of the shape generators and the shape space. That is, the input-output pairs of the training data set are formulated by the characteristic set and the shape generators. The predicted output, the new shape generator is computed by the training dataset and learning model. We finally reconstruct the new shape generator to geometric body shape by a stable multiple-frequency Fourier method and numerically simulate some examples.

In our second topic, we are concerned with the three-dimensional elastic scattering coefficients (ESC) and the enhancement of the elastic near cloaking. We establish the ESC of arbitrary three-dimensional objects and some of their properties using the elements of the elastic layer potential theory and multipolar expansions. We then construct the enhanced near elastic cloaking at a fixed frequency by using the ESC-vanishing-structures and transformation-elastodyamics. We also study some numerical examples on three-dimensional ESC.

In our third topic, we are concerned with the inverse problem of identifying magnetic anomalies with varying parameters beneath the Earth using geomagnetic monitoring. We study the information about the anomalies as well as their variations by the observations of the change in Earth's magnetic field, so called the secular variation. We rigorously establish the unique recovery results for this magnetic anomaly detection problem. We show that one can uniquely recover the locations, the variation parameters including the growth or decaying rates as well as their material parameters of the anomalies.

Table of Contents

List of Figures

List of Tables

List of Algorithms