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Quasi-Plus Sampling Edge Correction for Spatial Point Patterns

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Abstract

A widely applicable edge correction method for estimating summary statistics of a spatial point pattern is proposed. We reconstruct point patterns in a larger region containing the sampling window by matching sampled and simulated k th nearest neighbour distance distributions of the given pattern and then apply plus sampling. Simulation studies show that this approach, called quasi-plus sampling, gives estimates with smaller root mean squared errors than estimates obtained by using other popular edge corrections. We apply the proposed approach to real data and yield an estimate of a summary statistic that is more plausible than that obtained by a popular edge correction.

Keywords: edge effects, extrapolation, reconstruction, spatial point processes.

Running title: Quasi-plus sampling

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1 Introduction

Consider a stationary spatial point process Φ , which operates in the whole Euclidean space \mathbb{R}^d . Estimation of summary statistics of Φ based on a realisation φ observed in a bounded window W is usually complicated by edge effects. Unweighted empirical distributions are usually biased; edge effect correction methods, or simply edge corrections, have to be employed in order to get better estimates.

For the estimation of some distance distributions, such as the k th nearest neighbour distance distribution D_k , the empty space function F and Ripley's K -function, the so-called minus sampling is usually a valid edge correction. For ease of presentation we use $D = D_1$ as an example to illustrate various edge corrections. The idea of minus sampling is to consider the eroded window $W_{\ominus r} = \{x \in W : b(x, r) \subseteq W\}$, where $b(x, r)$ is the closed ball centred at x with radius r , so that whether or not a point lying in the eroded window has at least one r -close neighbour is known. Thus, the sample proportion of points in the eroded window having an r -close neighbour is a pointwise ratio-unbiased estimator of $D(r)$, called reduced-sample estimator.

On the other hand, there exists in the literature the so-called plus sampling. In this method D is estimated by the empirical distribution function of the sampled nearest neighbour distances of all points in W , assuming that the true nearest neighbour distance of each point in W can be measured, even if the true nearest neighbour is not located in W . This approach in fact requires that the edge effects do not exist at all. Thus, plus-sampling is strictly speaking not feasible because we require more information than that can be obtained

from the observation.

In this paper we introduce a quasi-plus sampling. We reconstruct points in a larger rectangular region containing the window W according to certain statistical rules and then we apply the technique of plus sampling. In particular, we consider the reconstruction procedure suggested in Tscheschel and Stoyan (2006), and compare the quasi-plus sampling based on reconstructed patterns with other edge corrections. Another possible nontrivial reconstruction in a case of specific Markov point process is the extrapolation obtained by adaptive coupling from the past (van Lieshout and Baddeley, 2002). We use Tscheschel and Stoyan’s reconstruction here because their algorithm can be applied to any empirical point patterns without any model assumptions. The forest data described in Section 6 shows that the proposed approach yields an estimate of Ripley’s K -function that is quite different from but more plausible than that obtained by the Ripley–Ohser isotropic edge correction.

2 Other edge corrections

Suppose the sampled nearest neighbour distance of point x_i is s_i and the distance from point x_i to the nearest boundary of the window is c_i . Baddeley and Gill (1997) considered the edge effects of a right censoring problem, because the sampled distance s_i is right-censored if $s_i > c_i$, and they proposed to use the Kaplan–Meier estimator,

$$\hat{D}^{KM}(r) = 1 - \prod_{\{i: s_i \leq r\}} \left(1 - \frac{\#\{i : s_i = s, s_i \leq c_i\}}{\#\{i : s_i \geq s, c_i \geq s\}} \right).$$

Another edge correction is to use weighted empirical distributions, which can be considered as Horvitz–Thompson style estimators (Baddeley, 1999). For example, an *ad hoc*

ratio-unbiased estimator for $D(r)$ was introduced by Hanisch (1984):

$$\hat{D}^H(r) = \frac{\sum_i v_i \mathbf{1}(s_i \leq c_i) \mathbf{1}(s_i \leq r)}{\sum_i v_i \mathbf{1}(s_i \leq c_i)},$$

where v_i is the reciprocal of the volume of the eroded window $W_{\ominus s_i}$. The denominator of $\hat{D}^H(r)$ is in fact an unbiased estimator of the intensity λ of the point process, whilst the numerator an unbiased estimator of $\lambda D(r)$.

These two types of estimator, as well as the reduced-sample estimator, also exist for D_k , F and K . Chiu and Stoyan (1998) showed theoretically the close relationship between the Hanisch estimator, the Kaplan–Meier estimator and the reduced-sample estimator. Consider the hazard rate of D ,

$$d\Lambda(r) = \frac{dD(r)}{1 - D(r)},$$

such that $D(r) = 1 - \exp\{-\int_0^r d\Lambda(s)\}$. The Kaplan–Meier estimator is constructed from the empirical hazard rate obtained by taking the ratio of the reduced-sample estimators of $dD(r)$ and $1 - D(r)$, assuming D has a density. The numerator of the Hanisch estimator is constructed from integrating the reduced-sample estimator of the density of λD . Stoyan (2006) demonstrated by simulation that for the estimation of D , the Hanisch estimator performs better than the Kaplan–Meier estimator and the reduced-sample estimator.

From the above formulae, we can see that the three estimators used only the uncensored s_i . To use further information from censored data, we may use imputation to handle point x_i for which $c_i < r < s_i$. For an individual point x_i , its empirical nearest neighbour distance

distribution is

$$\hat{D}_i(r) = \begin{cases} \mathbf{1}(r \geq s_i), & \text{if } s_i \leq c_i, \\ 0, & \text{if } r \leq c_i < s_i, \\ 1, & \text{if } r \geq s_i > c_i, \\ \text{unknown,} & \text{if } c_i < r < s_i, \end{cases}$$

where $\mathbf{1}(\cdot)$ is the indicator function. When $c_i < r < s_i$, the value of $\hat{D}_i(r)$ is unobservable but must be either 1 or 0, depending on whether there is an unobserved point of φ located in $b(x_i, r) \setminus W$ or not, respectively. Thus, if we knew the conditional probability $p_i(r) := \Pr(\#\{\Phi \cap b(x_i, r) \setminus W\} \geq 1 | \Phi \cap W = \varphi \cap W)$, then we might simulate a binary random variable X that is 1 with probability $p_i(r)$ and zero otherwise and impute the unknown value by X . It is, however, more natural to impute the unknown value by the mean of X , which is just $p_i(r)$, and then we may estimate D by the unweighted mean of \hat{D}_i :

$$\hat{D}^{\text{impute}}(r) = \frac{\sum_i \hat{D}_i(r)}{n},$$

where n is the number of points observed in W .

Doguwa and Upton (1990) estimated $p_i(r)$ by $1 - \exp(-\hat{\lambda}h_i)$, where $\hat{\lambda}$ is the intensity estimator and h_i is the volume of $b(x_i, r) \setminus W$. This estimator was derived from the assumption that Φ is a Poisson process. Reed and Howard (1997) proposed to use $\max(\hat{\lambda}h_i, 1)$ to estimate $p_i(r)$. Floresroux and Stein (1996) suggested to estimate $p_i(r)$ by the proportion, in the eroded window $W_{\ominus r}$, of analogous points having at least one r -close neighbour. A point x_j in $W_{\ominus r}$ is analogous to x_i if we shift the truncated ball $b(x_i, r) \cap W$ in such a way that the centre of the shifted ball is at x_j , there is no other point of φ lying in the shifted truncated

ball.

If the sampling window is rectangular, the imputation can be done by the toroidal correction so that the unknown value is $\mathbf{1}(r \geq s_i^*)$, where s_i^* is the distance from x_i to its nearest neighbour measured under periodic boundary conditions. This correction can be regarded as an example of quasi-plus sampling, because the toroidal correction can be viewed, under the stationarity assumption, as a naive reconstruction of the points in a $\underbrace{3 \times \cdots \times 3}_d$ grid of rectangles identical to W ; whenever we observed a point at x_i , we generate $3^d - 1$ more points so that we have 3^d points located at $x_i + (t_1, t_2, \dots, t_d)$, where t_j are 0 or \pm the length of the side of W parallel to the j th axis. In addition to the requirement of a rectangular window, a problem of such a reconstruction is that we may artificially introduce very small nearest neighbour distances that in fact do not exist in the underlying true process, e.g. a hard core process.

Imputing unknown values by conditional probabilities can also be regarded as a quasi-plus sampling; for each censored s_i , we pretend to simulate the individual distance of the nearest neighbour of x_i according to an estimate of the conditional probability $p_i(r)$. To determine the unknown value of the individual empirical $\hat{D}_i(r)$ for $c_i < r < s_i$, we do not use one simulated realisation to get a single zero-one value but take the theoretical mean, which is just the estimate of $p_i(r)$ itself. Except the summary function that is being estimated, i.e. the nearest neighbour distance distribution function in this example, other dependence relationships are, however, ignored in such an imputation method.

For a detailed discussion of edge corrections for spatial point patterns, see Illian *et al.*

(2008, Chapter 4).

3 Quasi-plus sampling by Tscheschel and Stoyan's reconstruction

The reconstruction procedure suggested in Tscheschel and Stoyan (2006) does not require any assumption on the shape of the window W , and we can specify a lot of dependence relationships in terms of summary statistics so that these relationships in the reconstructed pattern are very close to those in the observed points in W .

To reconstruct points in a rectangular $W_{\text{larger}} \supset W$, we use a method of conditional simulation with points in W fixed. We start with a realisation φ_0 of a binomial point process in $W_{\text{larger}} \setminus W$. The number of points is equal to the rounded value of the product of the estimated intensity and the volume of $W_{\text{larger}} \setminus W$. Then we generate a sequence $\{\varphi_0, \varphi_1, \varphi_2, \dots\}$ of patterns by the following rule: We randomly choose a point in φ_i and replace it by a new point that is independent and uniformly distributed in $W_{\text{larger}} \setminus W$; this new pattern is denoted by φ'_i . The successor φ_{i+1} is defined to be

$$\varphi_{i+1} = \begin{cases} \varphi'_i, & \text{if } E(\varphi'_i) \leq E(\varphi_i), \\ \varphi_i, & \text{otherwise,} \end{cases}$$

where $E(\varphi_i)$ is the sum of the discrepancies between the estimates of D_k based on the observed points only and based on the observed points together with the reconstructed

points in φ_i , i.e.

$$E(\varphi_i) = \sum_{k=1}^M \int_0^{R_k} \left\{ \hat{D}_k(r; \varphi^W, W) - \hat{D}_k(r; \varphi_i \cup \varphi^W, W_{\text{larger}}) \right\}^2 dr, \quad (1)$$

in which $\varphi^W = \varphi \cap W$, $\hat{D}_k(r; \varphi^W, W)$ is an estimate of $D_k(r)$ based on a finite point process φ^W observed in the window W , and M and R_k are suitably user-chosen upper limits. We use minus sampling for the estimates $\hat{D}_k(r; \varphi^W, W)$ and periodic boundary conditions for $\hat{D}_k(r; \varphi_i \cup \varphi^W, W_{\text{larger}})$ in the calculation of $E(\varphi_i)$. Although we may include other summary functions in the definition of $E(\varphi_i)$, Tscheschel and Stoyan (2006) showed that the functions D_k are quite sufficient for a meticulous and efficient reconstruction (note also that $\lambda K(r) = \sum_{k=0}^{\infty} D_k(r)$) and we follow their recommendation.

The iteration step from φ_i to φ_{i+1} is carried out until either a maximum number of iterations have been reached or at least s iterations have been performed and $E(\varphi_{i-s}) - E(\varphi_i) < \varepsilon$ for some small $\varepsilon > 0$ and big s .

A fine tuning of the above reconstruction algorithm is to consider an iterative procedure where in each step the estimates $\hat{D}_k(r; \varphi^W, W)$ in $E(\varphi_i)$ are obtained by the quasi-plus sampling in the previous step. However, we do not pursue such a fine tuning in this paper.

4 Adapted distance dependent intensity estimator

The quasi-plus sampling gives a natural estimator for $\lambda K(r)$, namely, the sample mean number of further points within distance r of a point. To get an estimator for $K(r)$, we still need an estimator of the intensity λ . In the simulation below we propose a new adapted distance dependent intensity estimator in the spirit of Stoyan and Stoyan (2000). The

estimator we used is

$$\begin{aligned}\hat{\lambda}(r) &= \frac{1}{\text{vol}(W)} \int_W \frac{\#(\Phi^* \cap b(x, r))}{\omega_d r^d} dx \\ &= \frac{1}{\text{vol}(W) \omega_d r^d} \sum_{y \in \Phi^*} \text{vol}(b(y, r) \cap W),\end{aligned}\tag{2}$$

where ω_d is the volume of a unit ball in \mathbb{R}^d and Φ^* the union of the true process Φ in the window W and the reconstructed pattern in $W_{\text{large}} \setminus W$, in which $W_{\text{large}} \supset W \oplus b(o, r)$ is presumed. This estimator is unbiased if $\Phi^* = \Phi$.

The advantages of using adapted distance dependent intensity estimators in the estimation of K have been discussed in Stoyan and Stoyan (2000), whose volume weighted intensity estimator was also used in the our simulation study below for the estimation of K under the translational correction.

5 Simulation

The quasi-plus sampling in this section used $M = 8$ and $R_k = 0.25$ in equation (1) and 100,000 independent realisations in a unit square were simulated for each model. If the number of points in a given pattern is small, there may exist an $M_1 < M$ such that we may not be able estimate $D_k(r)$ for $k > M_1$, and in this case we would let $\hat{D}_k(r) = 0$ for $k > M_1$. We discarded simulated realisations with 0 or 1 points. For each given point pattern with 2 or more points, we repeated the reconstruction $m = 5$ times in $W_{\text{large}} = [-R_k, 1 + R_k]^2$, and took the sample mean of the m estimates of the summary statistic being considered. In particular, we report the estimation results for the nearest neighbour distance distribution

function D , the empty space function F and Ripley's K -function. The number of iterations for each reconstruction was 1000 times the number of reconstructed points. We also would like to remark that although the reconstruction seems computationally intensive, for the examples we considered below, each reconstruction could be done in at most a few seconds on a Pentium IV 2.4GHz desktop computer.

Figure 1 shows the root mean squared errors in the estimation of D for processes sampled in a unit square. In each panel the upper three lines correspond to processes with $\lambda = 25$ and the lower three correspond to processes with $\lambda = 100$. Figure 1(a) is the result of Poisson processes. We can see that the quasi-plus sampling estimator is uniformly better than the Hanisch estimator or the Kaplan–Meier estimator over the whole range of r from 0 to 0.25, and this conclusion is also true for the Matérn type II hard core processes with hard core radius 0.05 and parent intensity $-\log(1 - 0.05^2\pi\lambda)/(0.05^2\pi)$. For the Matérn cluster processes with cluster radius 0.1 and daughter intensity 5, the quasi-plus sampling estimator is uniformly better than the Kaplan–Meier estimator, and is much better than the Hanisch estimator for $r \leq 0.1$ and only slightly worse than it when $r \geq 0.15$.

Figure 1 about here

Figure 2 shows the bias in the estimation of D . The upper three lines correspond to the case that $\lambda = 100$, whilst the lower three lines $\lambda = 25$. The bias of the quasi-plus sampling is not worse than that of the Hanisch estimator and is better than the Kaplan–Meier estimator. For the Matérn cluster process, however, the quasi-plus sampling is the worst and for the hard core process, the quasi-plus sampling has a positive bias for small r because we may

introduce nearest neighbour distances that are shorter than the hard core distance. This is the price we have to pay for any nonparametric reconstruction. Of course if we know we are reconstructing a hard core process, we can enforce such a hard core distance between all points.

Figure 2 about here

For the estimation of F , we still compare the quasi-plus sampling estimator with the Kaplan–Meier estimator (Baddeley and Gill, 1997)

$$\hat{F}^{KM}(r) = 1 - \exp \left\{ - \int_W \frac{\mathbf{1}(s_x \leq c_x) \mathbf{1}(s_x \leq r)}{\text{vol}(W_{\ominus s_x} \setminus \cup_{x_i \in \Phi} b(x_i, s_x))} dx \right\}$$

and the Hanisch estimator (Chiu and Stoyan, 1998)

$$\hat{F}^H(r) = \int_W \frac{\mathbf{1}(s_x \leq c_x) \mathbf{1}(s_x \leq r)}{\text{vol}(W_{\ominus s_x})} dx,$$

where s_x is the distance from x to the nearest member of Φ and c_x is the shortest distance from x to the boundary of W . The same as in the estimation of D , the quasi-plus sampling has the least root mean squared errors for the three processes considered (see Figure 3). The bias caused by the quasi-plus sampling, though worse than the other two, is not really big; the maximum absolute bias is about 0.005 only for the Poisson process with $\lambda = 25$, and the cluster and hard core processes give similar maximum biases.

Figure 3 about here

For the estimation of K , we use $\hat{\lambda}(r)$ in equation (2) as the intensity estimator in quasi-plus sampling. The competitors are the isotropic edge corrected estimator for $\lambda^2 K(r)$ with

$1/\{n(n-1)\}$ as the estimator for λ^{-2} :

$$\hat{K}^{\text{iso}}(r) = \frac{1}{n(n-1)} \sum_{\substack{x,y \in \Phi \cap W \\ x \neq y}} \frac{\mathbf{1}(\|y-x\| \leq r)k(x,y)}{\text{vol}(\{x \in W : \partial b(x, \|y-x\|) \cap W \neq \emptyset\})},$$

where $\partial b(x, r)$ is the boundary of $b(x, r)$ and $k(x, y)$ is equal to 2π divided by the sum of all angles of the arcs in $\partial b(x, \|x-y\|) \cap W$ (Stoyan, Kendall and Mecke, 1995, pp. 134-135), and the translational edge corrected estimator for $\lambda^2 K(r)$ with the volume weighted adapted distance dependent intensity estimator suggested by Stoyan and Stoyan (2000)

$$\hat{K}^{\text{trans}}(r) = \frac{1}{\hat{\lambda}_v(r)^2} \sum_{\substack{x,y \in \Phi \cap W \\ x \neq y}} \frac{\mathbf{1}(\|y-x\| \leq r)}{\text{vol}(W_x \cap W_y)},$$

where $W_x = W + x = \{w + x : w \in W\}$ and

$$\hat{\lambda}_v(r) = \left\{ \sum_{x \in \Phi \cap W} \frac{\text{vol}(W \cap b(x, r))}{\pi r^2 - \frac{8}{3}r^3 + \frac{1}{2}r^4} \right\}.$$

Figure 4 about here

From Figure 4, we can see that the quasi-plus sampling, again, works better, in terms of the root mean squared errors, than the isotropic correction and the translational correction with Stoyan and Stoyan's volume weighted intensity estimator. The isotropic correction gives an unbiased estimator for $K(r)$ of a Poisson process, but for the Matérn cluster process, the quasi-plus sampling has less bias than the isotropic correction and it is also the case for the hard core process with $\lambda = 100$.

Figure 4 shows the quasi-plus sampling estimated K up to not $r = R_k = 0.25$ but $r = 0.5$ obtained under periodic boundary conditions. For $\lambda = 25$, there are cusp points on the

estimates by the quasi-plus sampling in Figures 4(a) and (c) at $r = 0.25 = R_k$. The reason for the Poisson process case is clear: no reconstruction is better than any reconstruction according to the empirical dependence structure; when we match the sampled and simulated k th nearest neighbour distance distributions up to $R_k = 0.25$, we do not introduce interaction beyond $r > 0.25$ and so we have less bias and smaller root mean squared errors. We do not have a definite explanation for the cusp point in the hard core process case but obviously it is caused by the low intensity and the use of periodic boundary conditions. This also leads to another question in practice: up to what value of r should we stop reconstruction and use toroidal correction? If λ is small, our empirical results suggest that it may be better to have a small W_{larger} and then use toroidal correction, because the estimate $\hat{D}_k(r; \varphi \cap W)$ in equation (1) is not too reliable. Nevertheless, if λ is large, using solely a large W_{larger} or using a small W_{larger} combined with toroidal correction has more or less the same performance, and we do not observe any cusp points for $\lambda = 100$ in our simulation.

We obtained similar results and the same conclusions when some other parameter values were used for the Matérn cluster processes and the Matérn type II hard core processes.

We can conclude that no edge correction is uniformly best or worst in terms of the bias. However, if the root mean squared errors is the major criterion, then the quasi-plus sampling, using Tscheschel and Stoyan's reconstruction algorithm, is the winner in all the examples covered.

6 Real data

The central part of Figure 5 shows the locations of 70 trees (chestnut, *Castanea sativa* Mill) observed in a polygonal window, which is of size 13909.25m^2 and has an x -range of $[13.7, 174.6]$ and a y -range of $[6.6, 213.5]$, in the Vosges Mountains, France, embedded in the rectangular window $[-16.3, 204.6] \times [-23.4, 243.5]$ containing a reconstructed pattern using Tscheschel and Stoyan's algorithm, with $R_k = 30$ and $M = 8$.

Figure 5 about here

The estimates of the three summary statistics are given in Figure 6. For the quasi-plus sampling we repeated the reconstruction $m = 100$ times and considered two scenarios, namely, $M = 8$ and $M = 15$. We have to admit that the choice of M is arbitrary here and we recommend the interested reader to try different M , as we did below, to see if the estimates agree or not. Although we could obtain more information on the dependence between points by using a larger M , the precision of the estimation of D_k is getting worse when k is getting larger. Experience suggests that we may choose $5 \leq M \leq 40$, depending on how many points we observed and which summary statistics we would like to estimate.

For the estimation of D and F , because the information contained in D_k for large k may not be so relevant, we do not have any visually distinguishable differences between $M = 8$ and $M = 15$ in the quasi-plus sampling, and also do not have any practically meaningful differences between the quasi-plus sampling estimates and the Hanisch/Kaplan–Meier estimates. However, in the estimation of K , we notice that $M = 8$ and $M = 15$ gave quite different estimates. The reason may be as follows. When estimating $K(r)$, we have to

find, for point x_i , its own k_i such that the k_i th nearest neighbour is within distance r from x_i and the k_{i+1} th nearest neighbour is more than r away from x_i . Thus, the estimates of K obtained by using $M = 8$ and $M = 15$ may be quite different. It is reasonable to believe that the estimates obtained by using a larger M is more close to the truth, provided that we have sufficient number of observed points to give good estimates of D_k for $k \leq M$. Here, with 70 points, we choose the one obtained by using $M = 15$. However, the estimated K by using isotropic edge correction is also different from the one obtained by reconstruction with $M = 15$. The reason is that the biggest cluster in the pattern is close to the northwest boundary of the sampling window; the isotropic edge correction will assume that this cluster, because of edge effects, is just a part of an even bigger cluster and so as a result, the K -function estimated by the isotropic edge correction will be higher. On the other hand, we do not see any clusters than are bigger than the one near the northwest boundary, patterns reconstructed according to Tscheschel and Stoyan's algorithm are unlikely to contain much bigger clusters than the biggest one we observed. As a result, the estimated K obtained by using reconstruction with $M = 15$ is lying below the one obtained by the isotropic edge correction, and we believe that the former gives a better description of the second order properties of the given pattern than the latter, because we do not have any strong evidence to suggest the presence of bigger clusters.

Figure 6 about here

Acknowledgements

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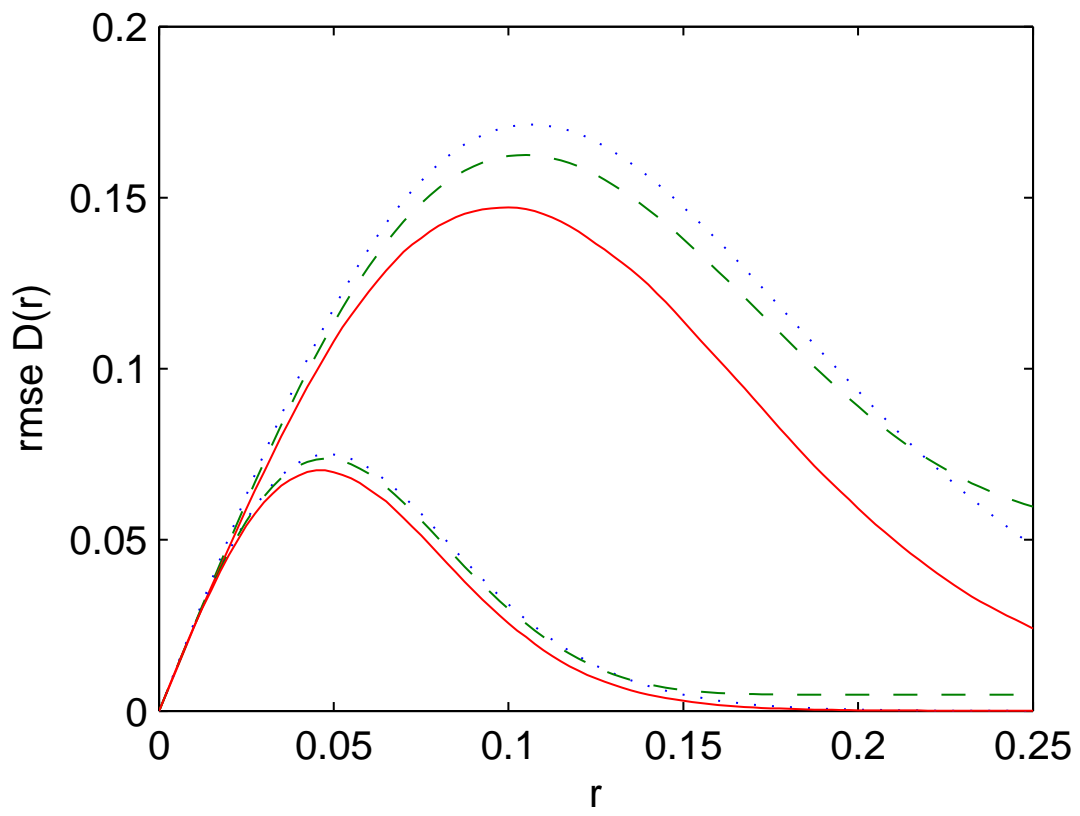


Figure 1(a)

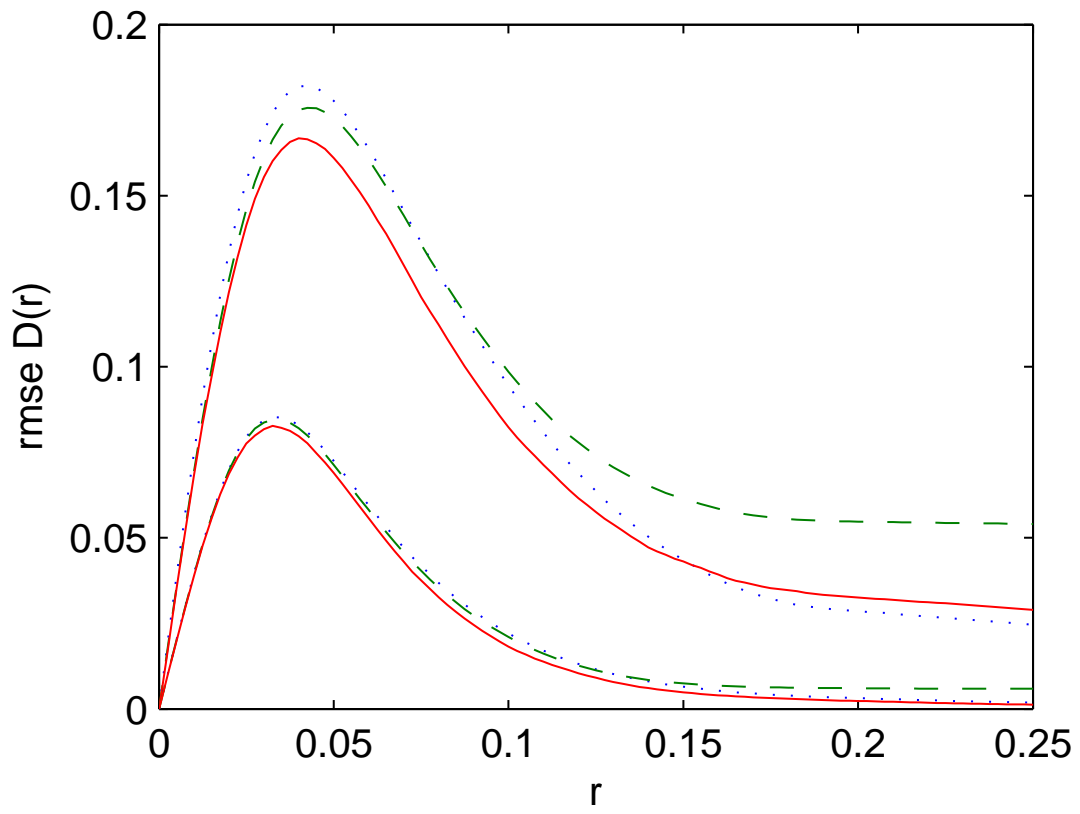


Figure 1(b)

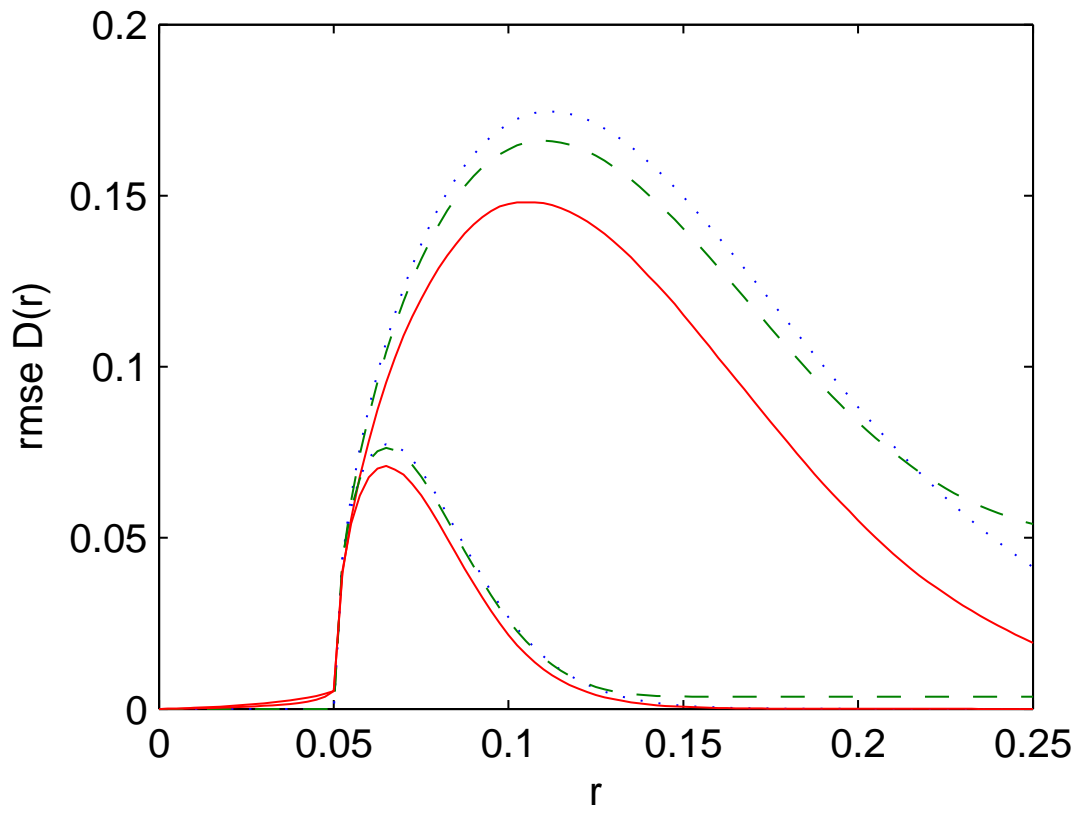


Figure 1(c)

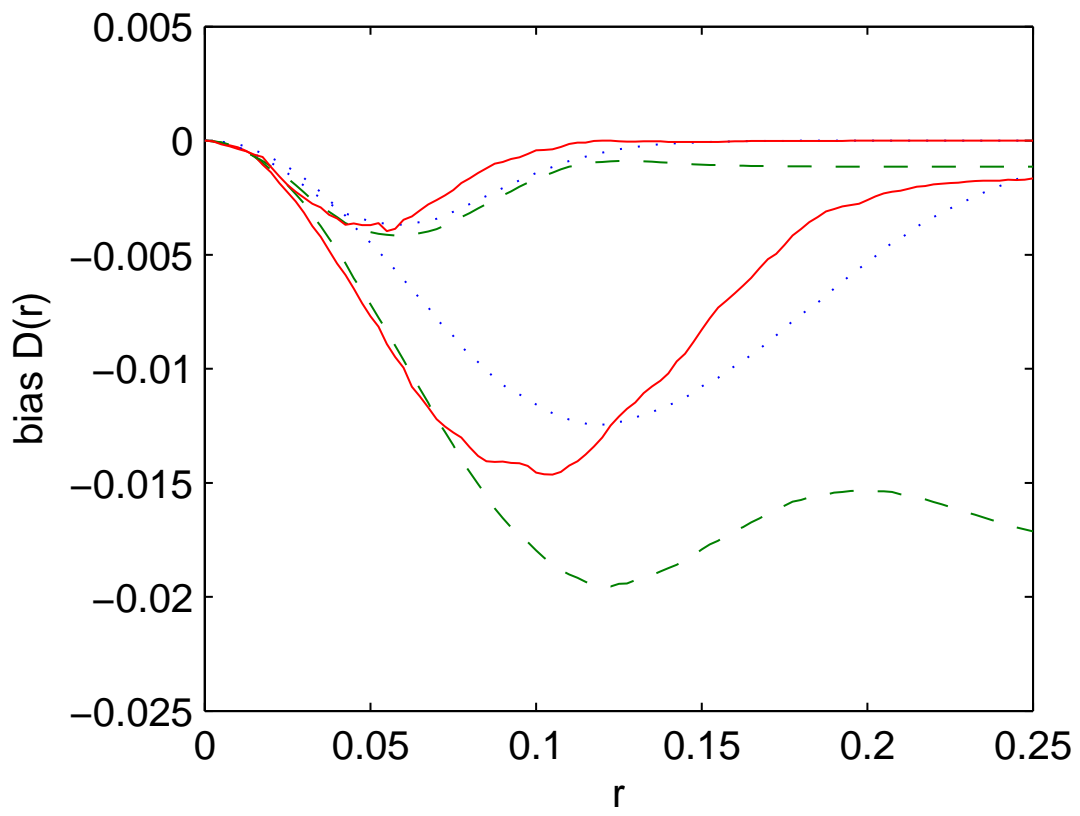


Figure 2(a)

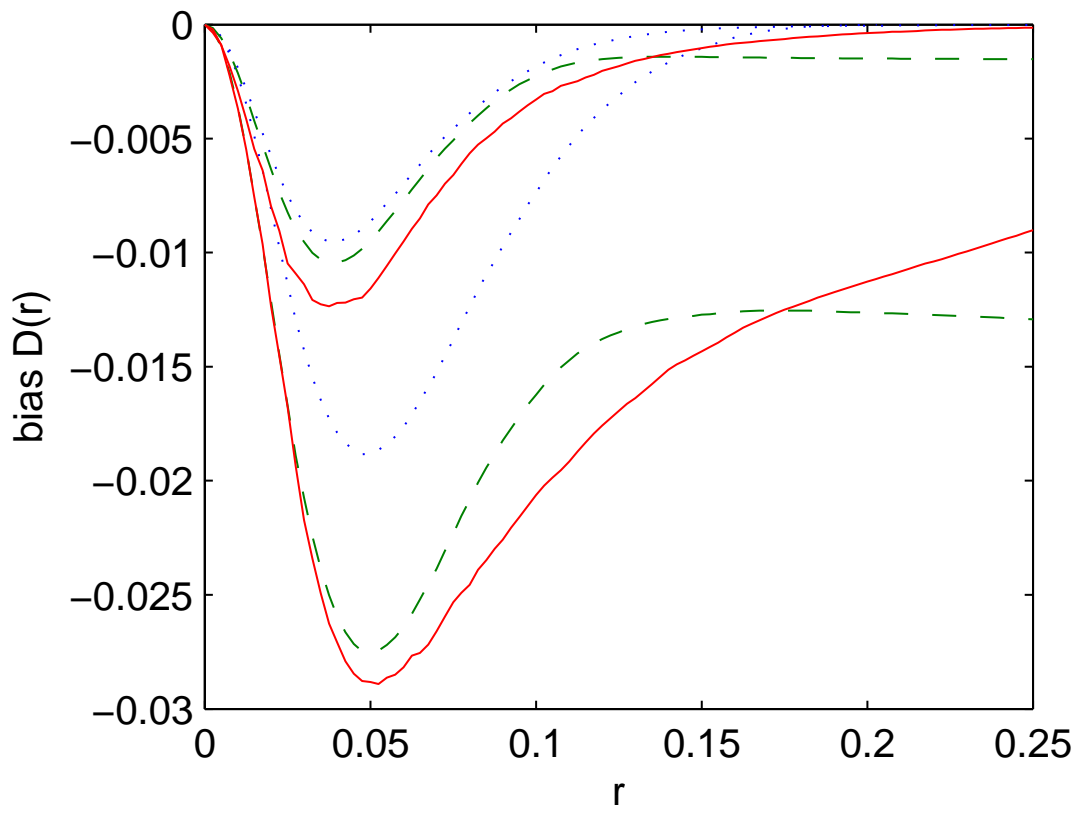


Figure 2(b)

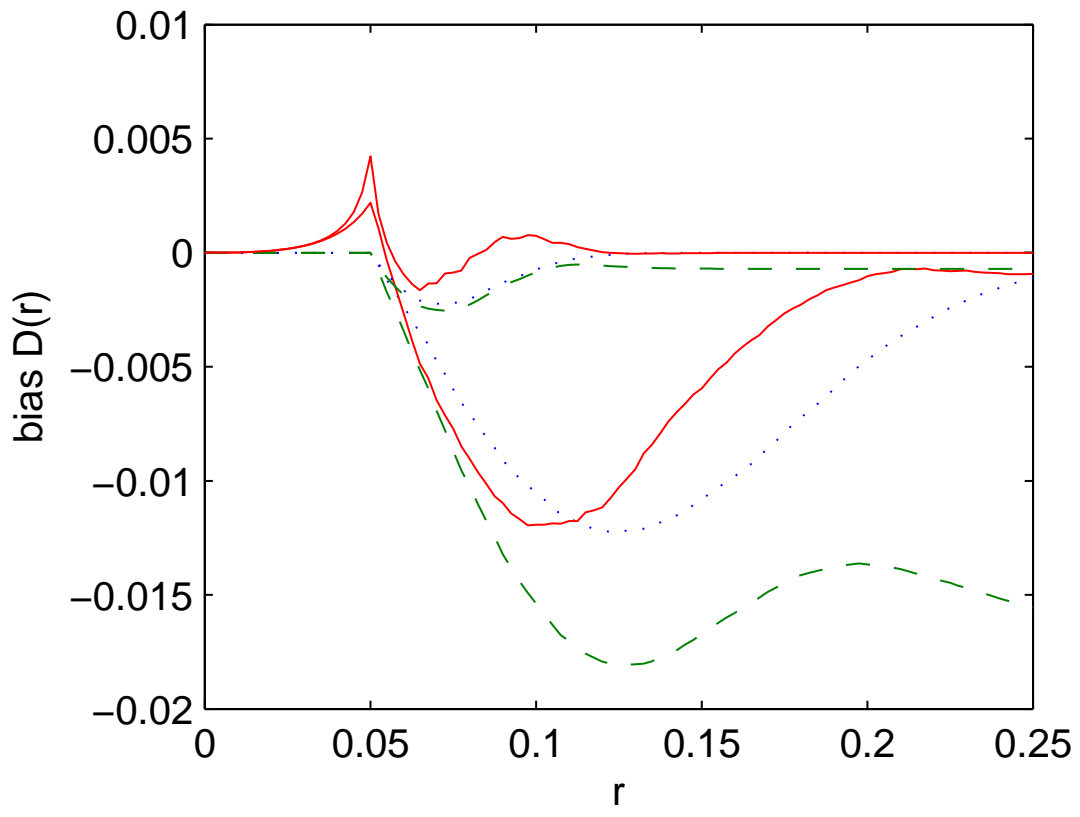


Figure 2(c)

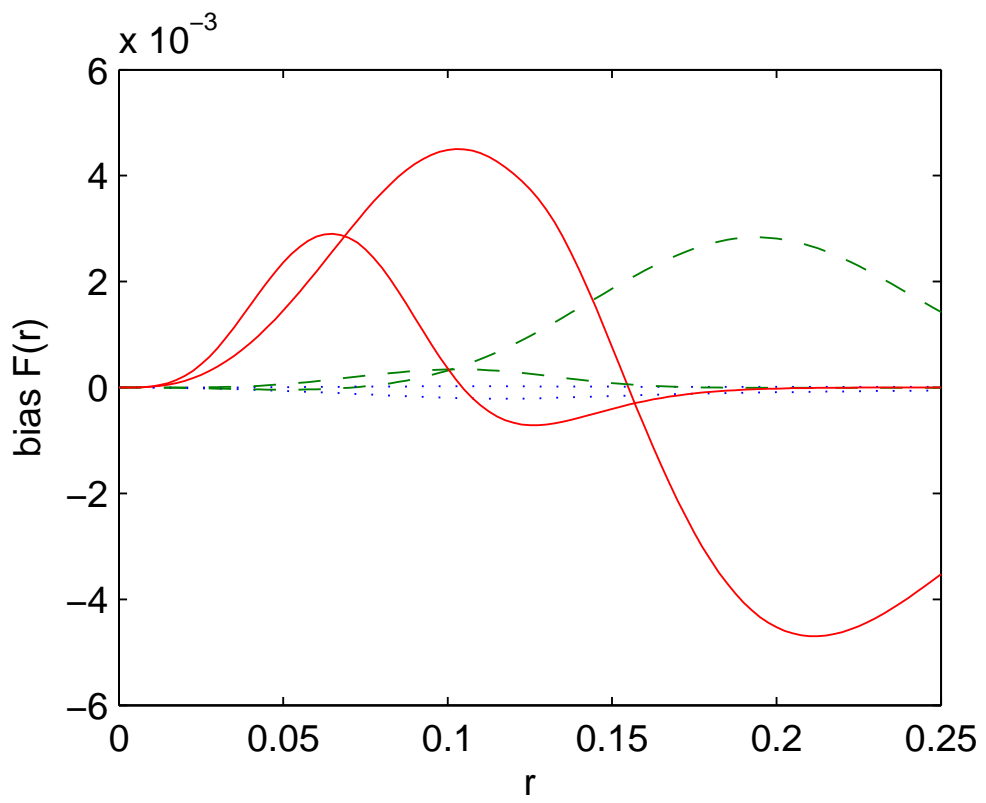
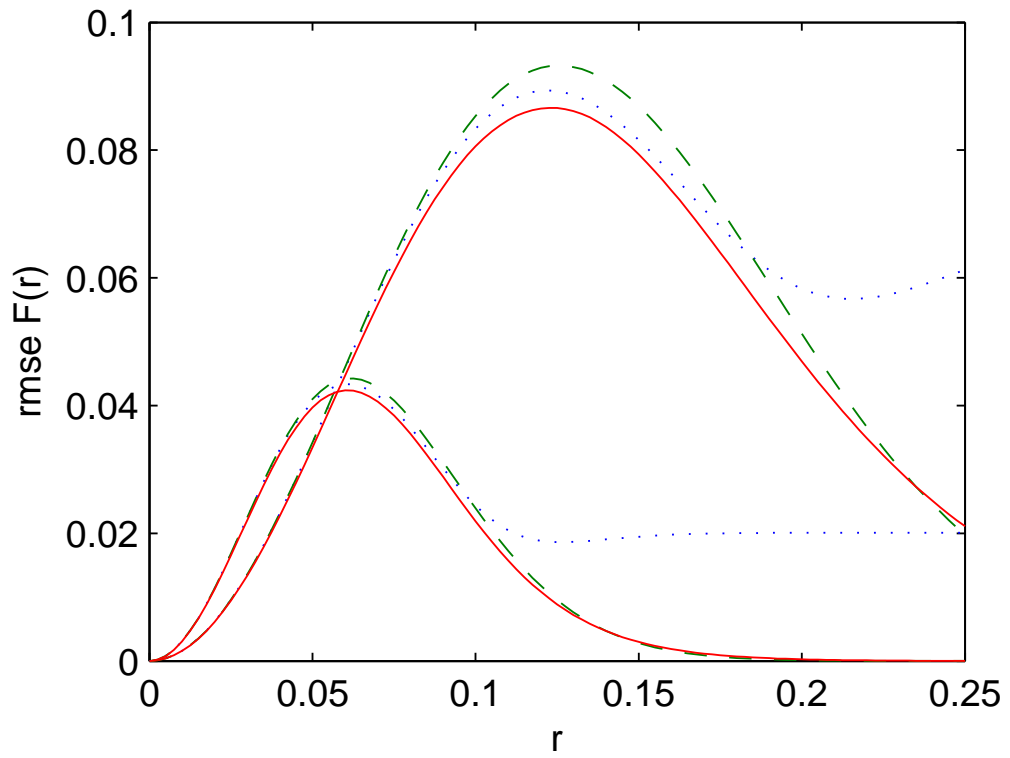


Figure 3(a)

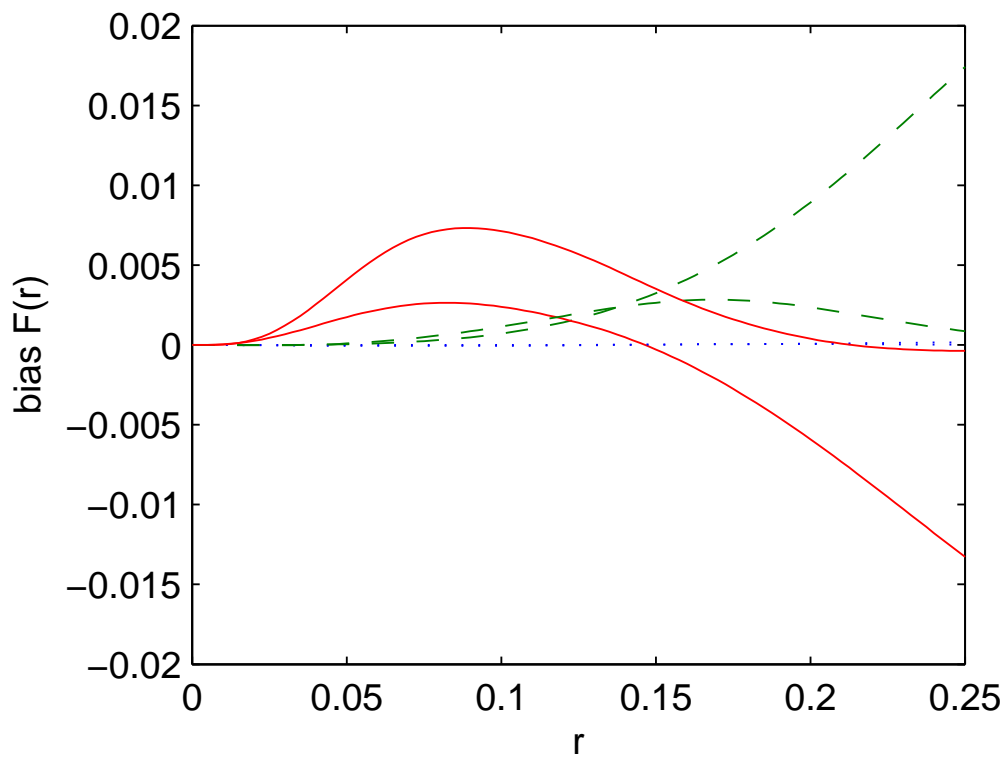
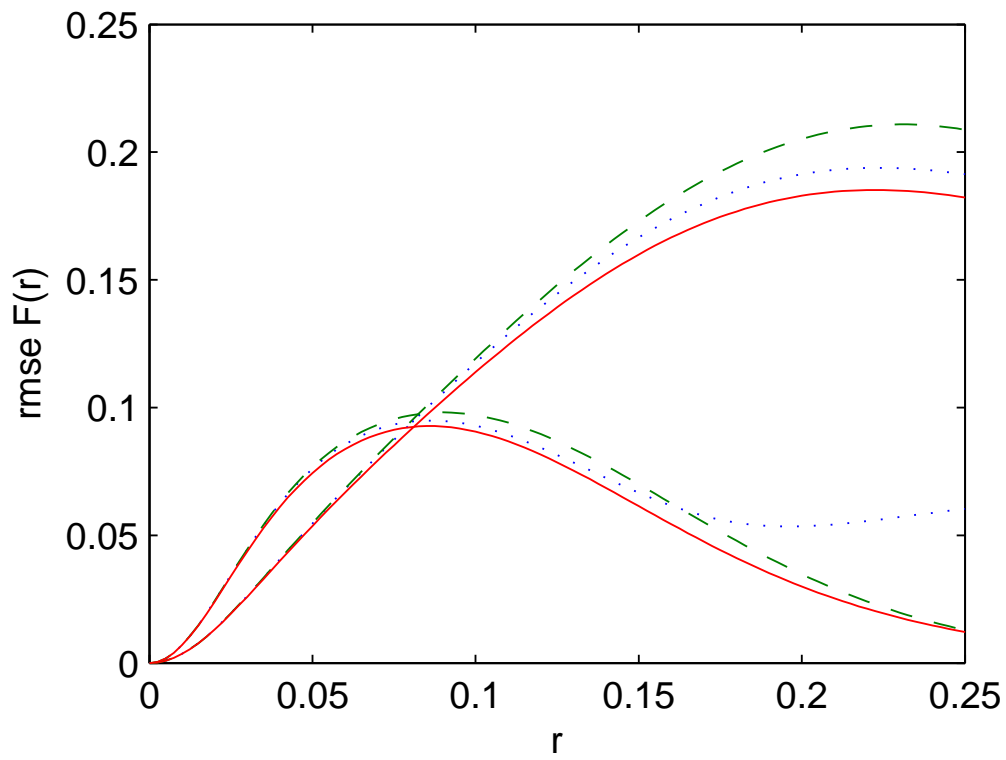


Figure 3(b)

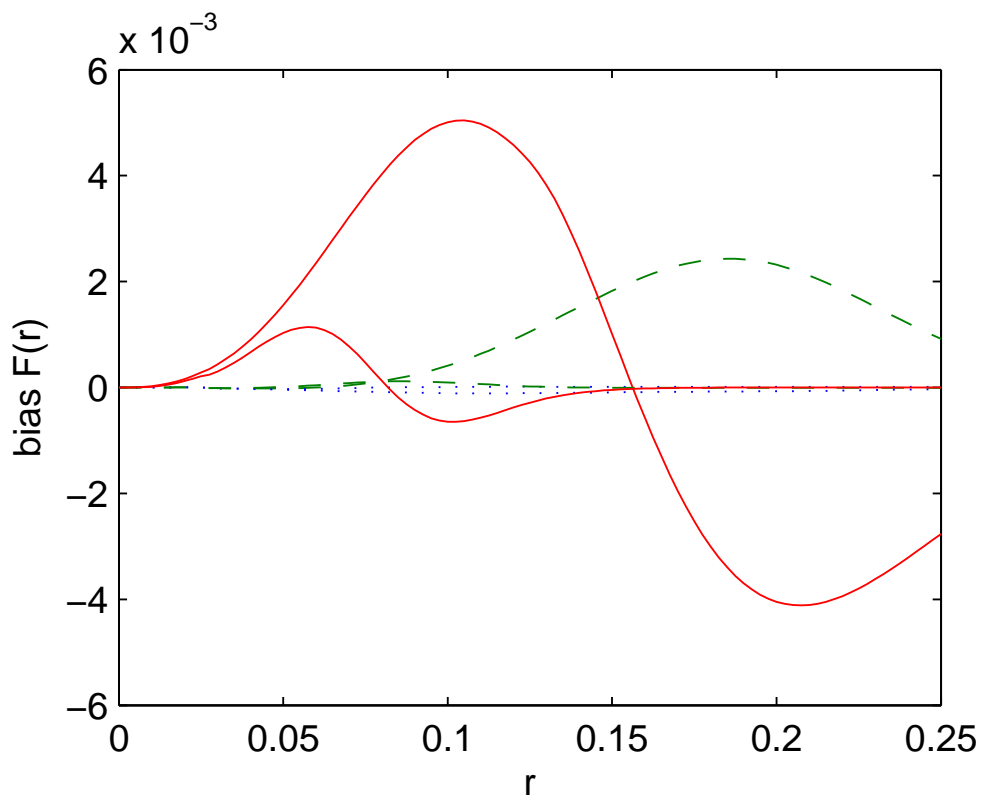
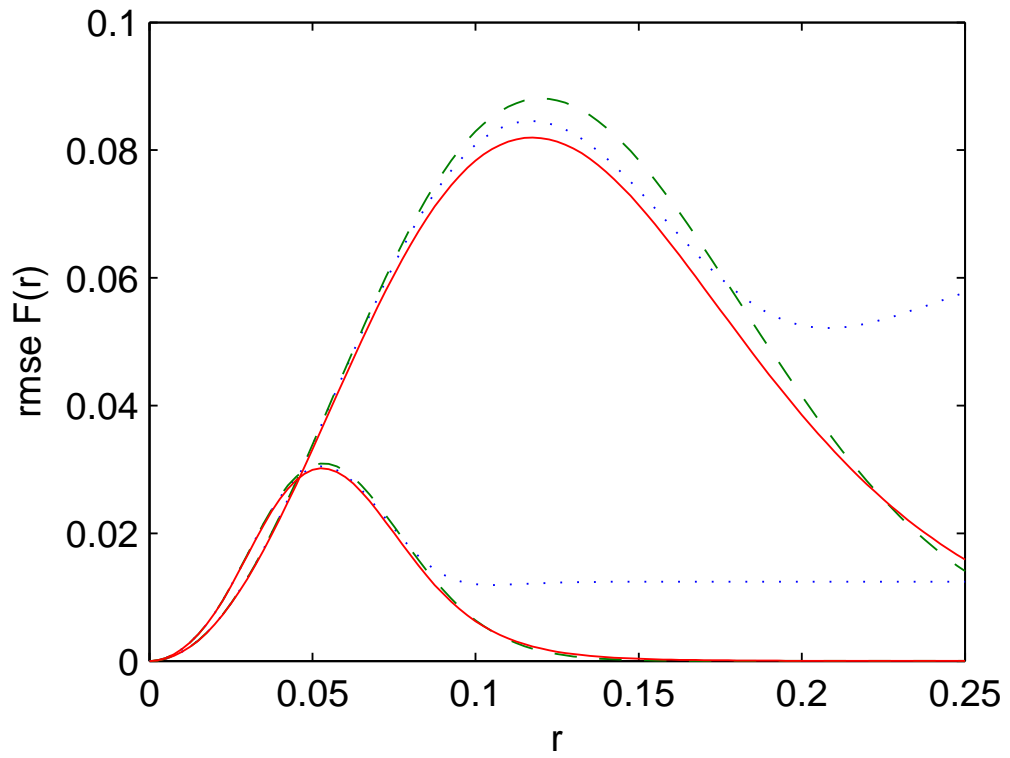


Figure 3(c)

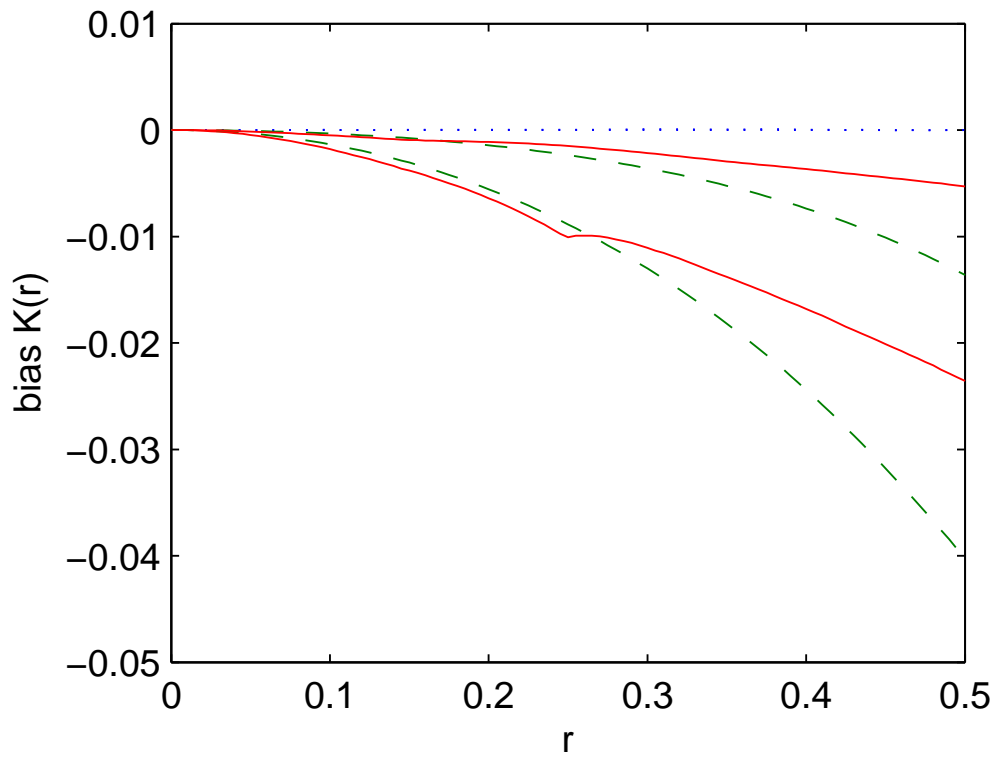
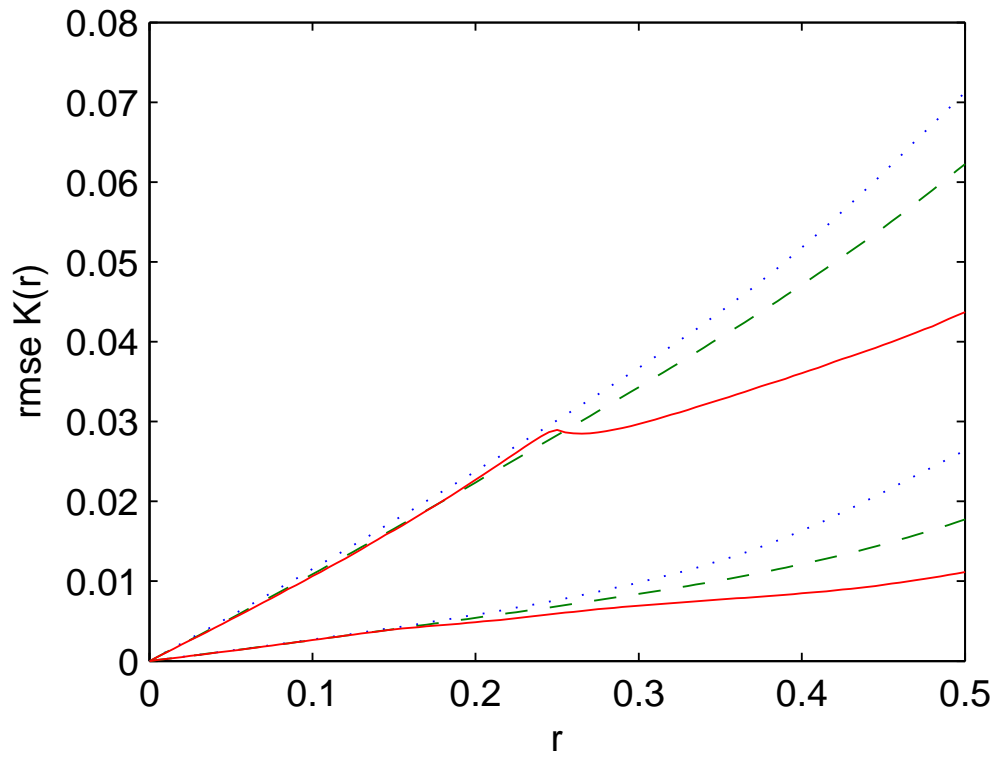


Figure 4(a)

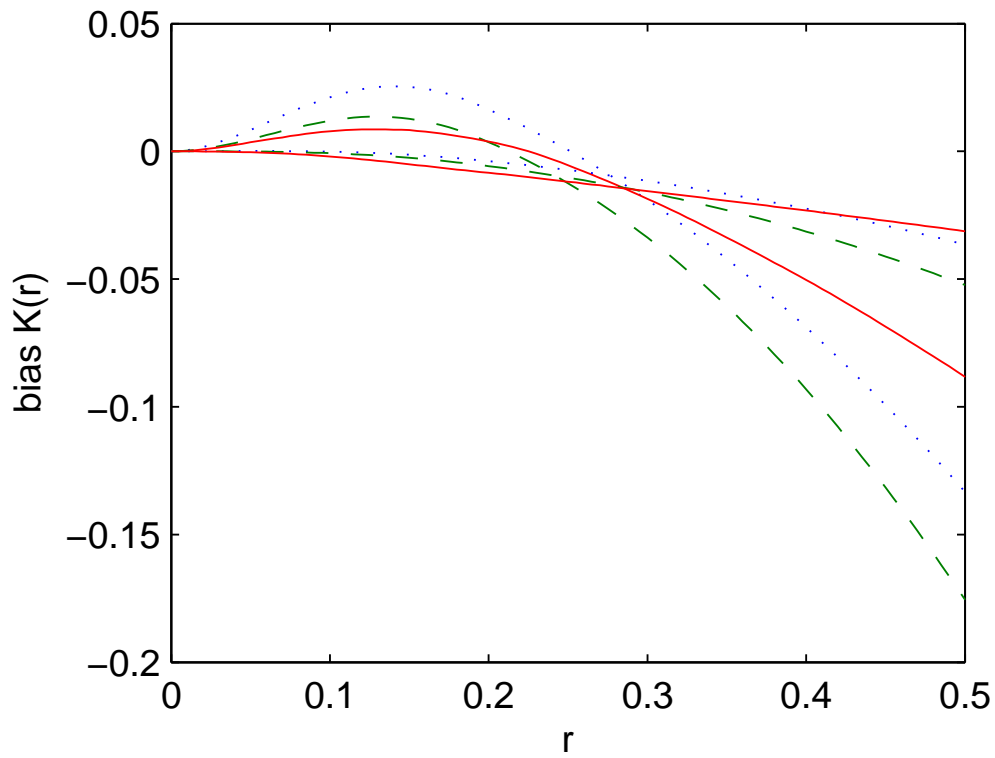
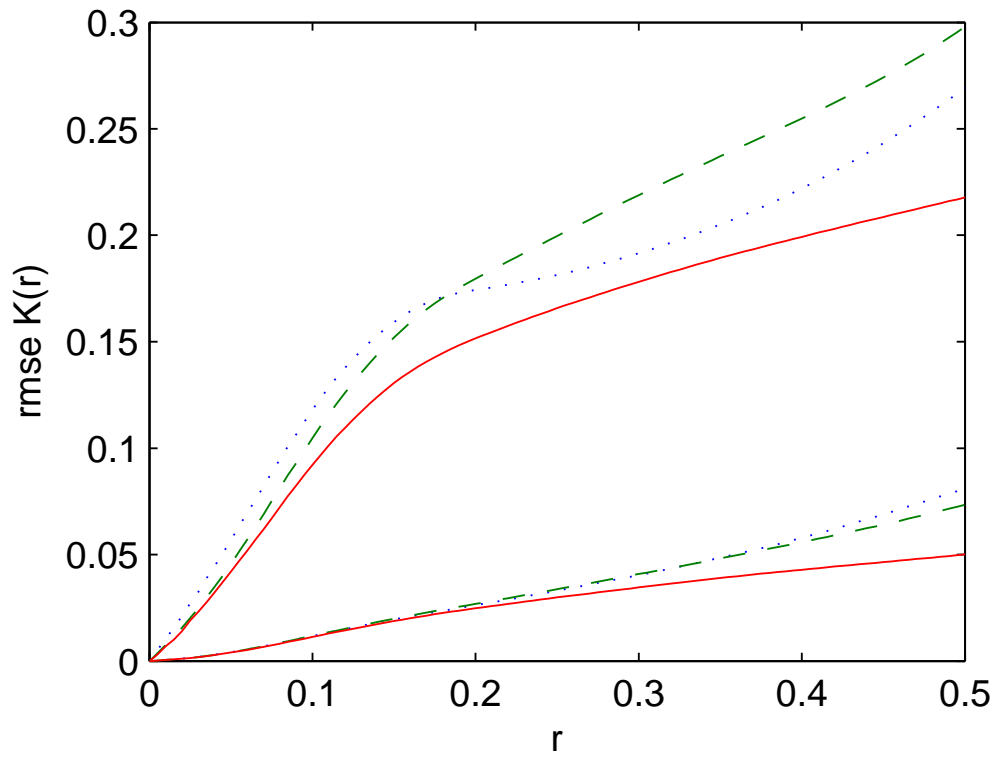


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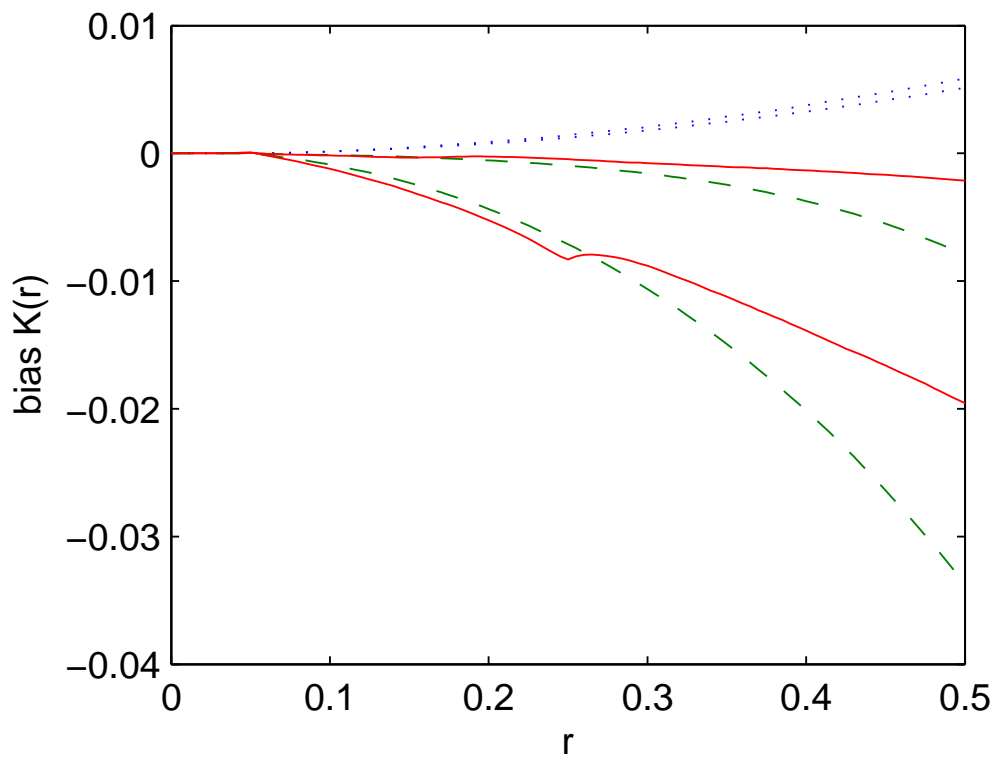
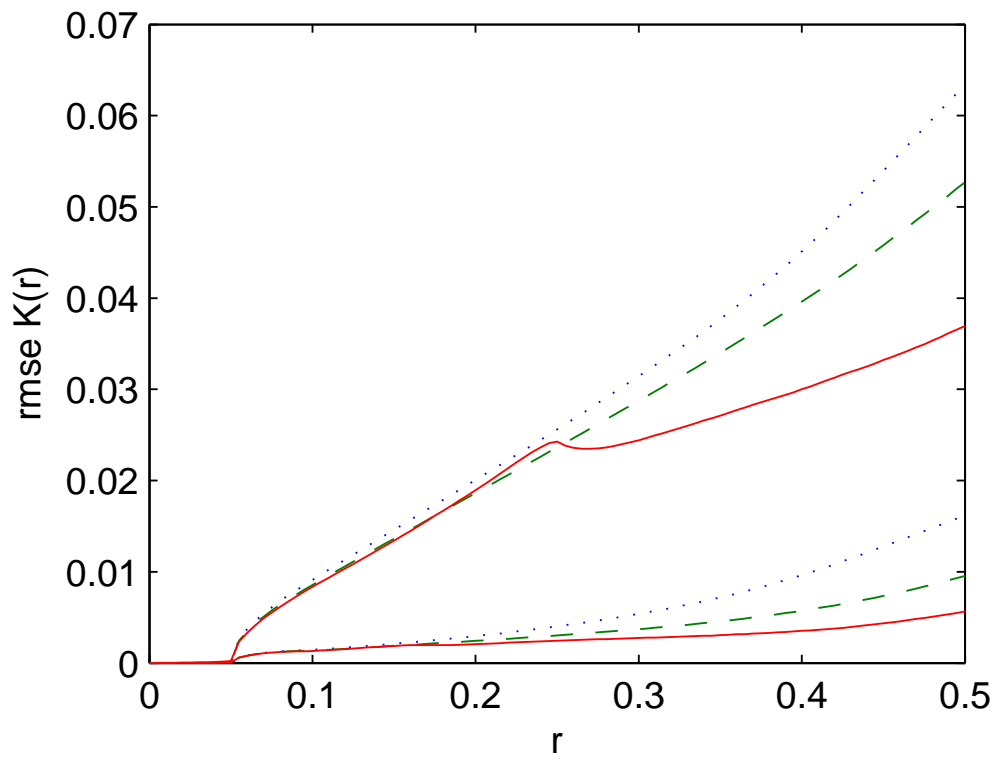


Figure 4(c)

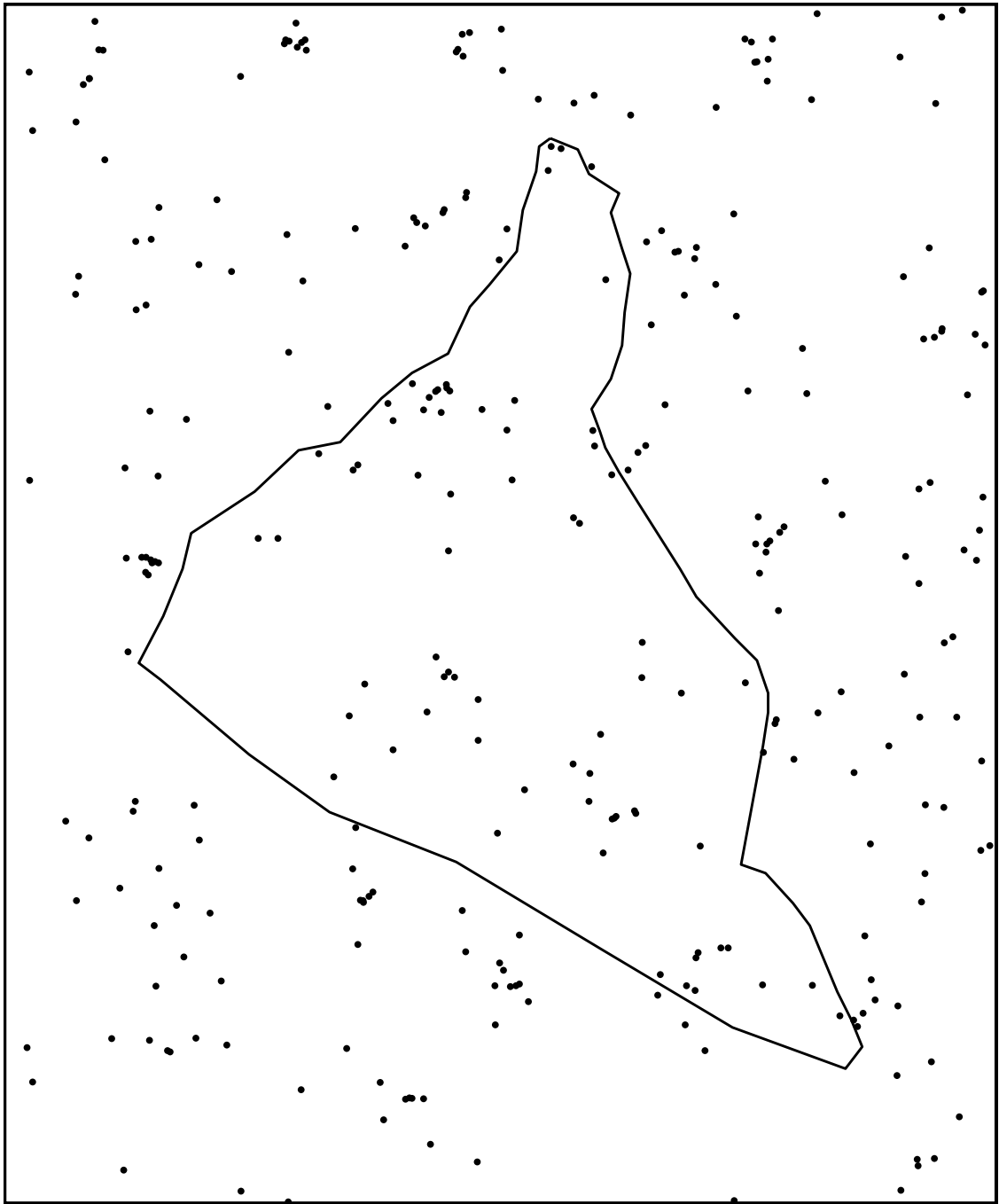


Figure 5

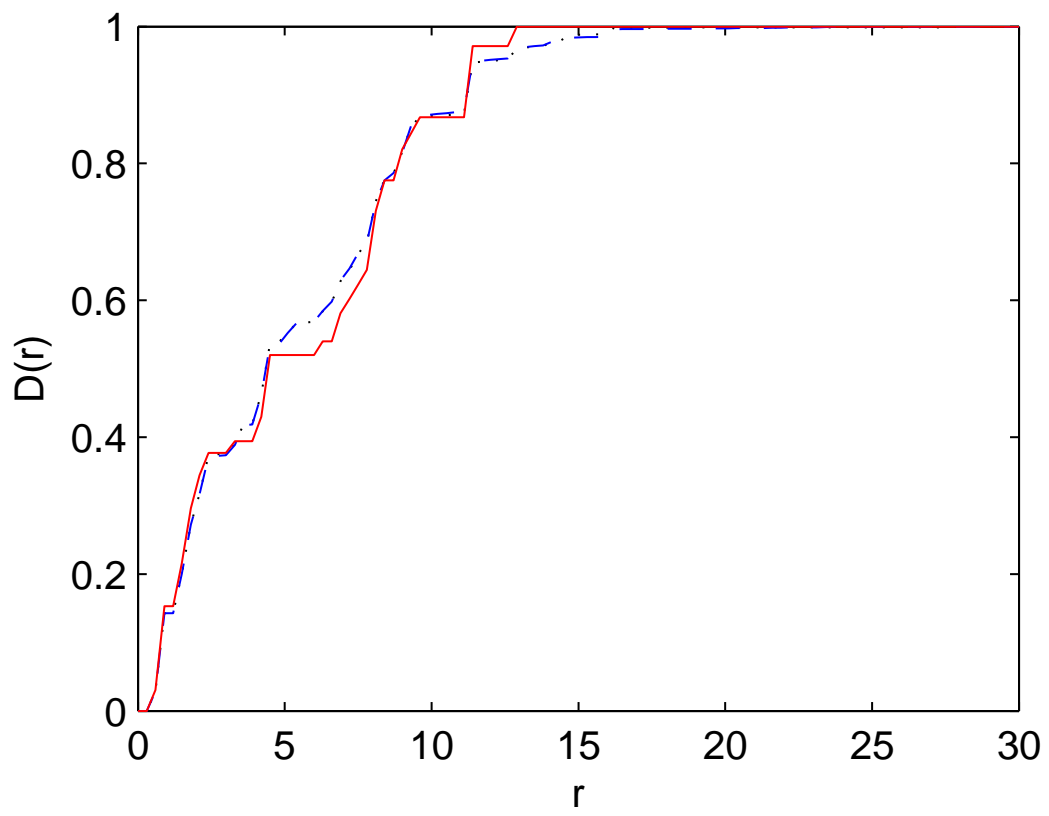


Figure 6(a)

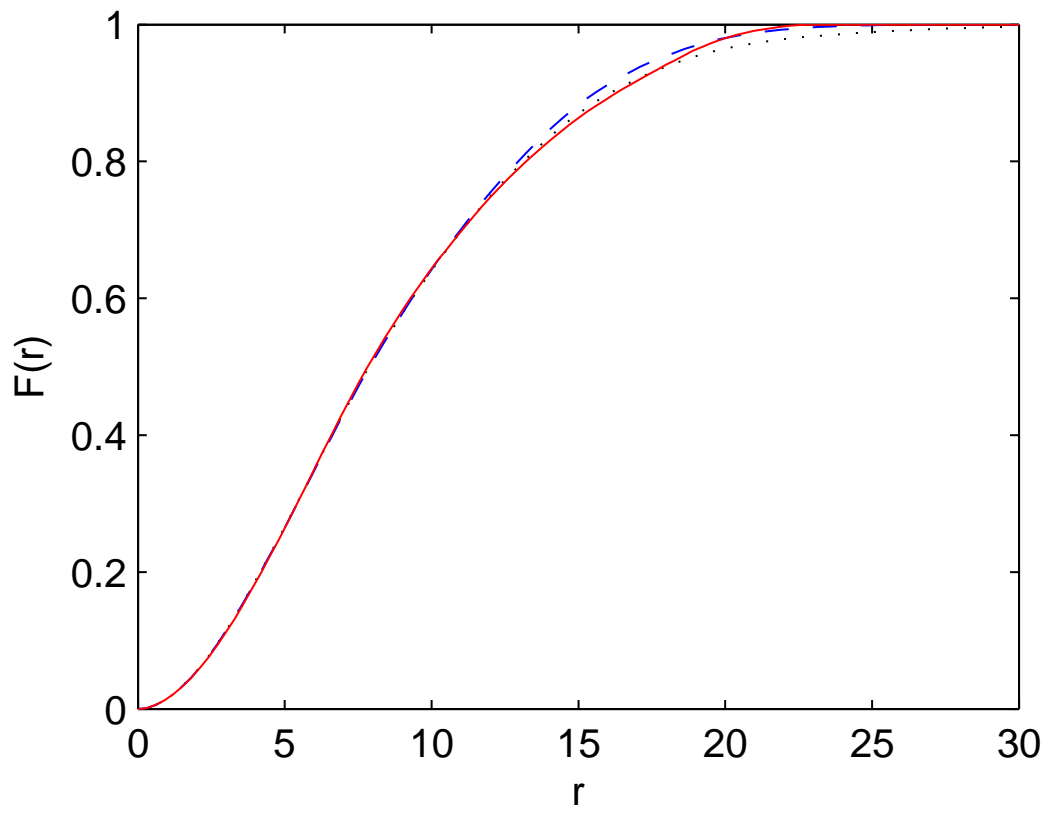


Figure 6(b)

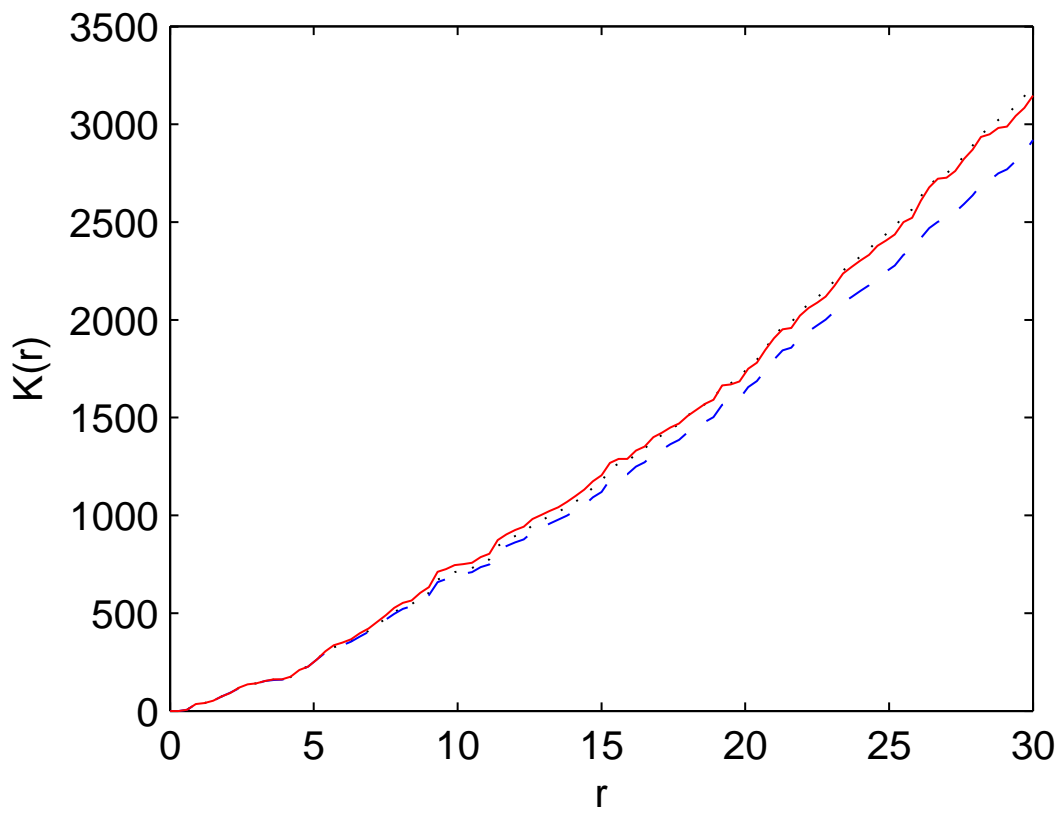


Figure 6(c)

List of figure captions

Figure 1: Root mean squared errors in the estimation of the nearest neighbour distance distribution function $D(r)$ of (a) Poisson processes, (b) Matérn cluster processes and (c) Matérn type II hard core processes. (— Quasi-plus sampling; \cdots Hanisch; - - - Kaplan-Meier).

Figure 2: Bias in the estimation of the nearest neighbour distance distribution function $D(r)$ of (a) Poisson processes, (b) Matérn cluster processes and (c) Matérn type II hard core processes. (— Quasi-plus sampling; \cdots Hanisch; - - - Kaplan-Meier).

Figure 3: Root mean squared errors (upper panel) and bias (lower panel) in the estimation of the empty space function $F(r)$ of (a) Poisson processes, (b) Matérn cluster processes and (c) Matérn type II hard core processes. (— Quasi-plus sampling; \cdots Hanisch; - - - Kaplan-Meier).

Figure 4: Root mean squared errors (the upper row) and bias (the lower row) in the estimation of Ripley's reduced second order moment function $K(r)$ of (a) Poisson processes, (b) Matérn cluster processes and (c) Matérn type II hard core processes. (— Quasi-plus sampling; \cdots isotropic correction; - - - translational correction).

Figure 5: The polygonal window in the central part contains the locations of 70 trees (chestnut, *Castanea sativa* Mill) in the Vosges Mountains, France. Points outside the polygonal window are 1 of 100 reconstructed patterns using Tscheschel and Stoyan's algorithm.

Figure 6: Estimates of the three summary statistics of the data given in Figure 5. (a) The estimates of D (— Hanisch; \cdots quasi-plus sampling with $m = 100$ and $M = 8$; - - - quasi-plus sampling with $m = 100$ and $M = 15$), (b) the estimates of F (— Kaplan–Meier; \cdots quasi-plus sampling with $m = 100$ and $M = 8$; - - - quasi-plus sampling with $m = 100$ and $M = 15$), and (c) the estimates of K (— isotropic edge correction with $1/\{n(n-1)\}$ as the estimator for λ^2 ; \cdots quasi-plus sampling with $m = 100$ and $M = 8$; - - - quasi-plus sampling with $m = 100$ and $M = 15$).