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# Chained Financial Contracts and Global Banks

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## Abstract

This paper studies a chained credit contract based on Hirakata, Sudo and Ueda (2013) in which investors lend funds to banks and banks lend to entrepreneurs in an imperfect financial market. We show that the optimality condition of this contract has a simple, symmetric structure analogous to the one in Bernanke, Gertler and Gilchrist (1999), and that the external finance premium is increasing in both the entrepreneurs' and the bank's capital to net worth ratio. We apply the chained credit contract to analyse global banks, and show that the common lender effect drives the positive comovement of the external finance premia across economies.

JEL Classification: E44.

Keywords: Financial accelerators; banks; chained credit contracts.

## 1 Introduction

Hirakata, Sudo and Ueda (2013) (Henceforth HSU) develop a chained credit contract based on the financial accelerator in Bernanke, Gertler and Gilchrist (1999) (Henceforth BGG). They study the optimal decision by banks which borrow from depositors with a loan contract and lend to entrepreneurs with another loan contract when there is a costly state verification problem. This paper shows that the solution of this contracting problem has a surprisingly simple structure which depends symmetrically on the cutoff values of the idiosyncratic productivity shocks in each of the two borrowers. Furthermore, the solution resembles the one in BGG. This allows us to show analytically that the external finance premium is increasing in both the entrepreneurs' and bank's capital to net worth ratio.

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We apply this chained credit contract framework to analyse global banks. Global banks take deposits from home and foreign investors and lend to home and foreign entrepreneurs. The solution of the optimal contract is one in which the external finance premia for home and foreign entrepreneurs have a common component which depends positively on the cutoff value for the global banks' idiosyncratic productivity shock. We show analytically and numerically that the common lender effect leads to positive comovement between external financial premia internationally, in line with Dedola and Lombardo (2009), Devereux and Yetman (2010), Ueda (2012), Kollmann, Enders and Muller (2011), Krugman (2008) and Davis (2014).

## 2 Chained Credit Contract

The financial contracts involve investors, banks and entrepreneurs. Banks take deposits from investors and lend to entrepreneurs. Both entrepreneurs and banks face idiosyncratic shocks to their return to capital. Denote the realised return of a borrower  $\omega^i R^i$ , where  $i \in \{E, F\}$ , where  $E$  and  $F$  denote entrepreneurs and banks.  $R^i$  is the average return of sector  $i$ , and  $\omega^i$  is an idiosyncratic shock for sector  $i$ .  $\omega^i$  is an *i.i.d* random variable,  $\log(\omega^i) \sim N(-0.5(\sigma^i)^2, (\sigma^i)^2)$  and  $E(\omega^i) = 1$ .  $\omega^E$  and  $\omega^F$  are independent.

Financial friction exists because the lender must pay a monitoring cost to observe the shock for a specific borrower, which destroys a fixed fraction  $\mu^i$  of the borrower's capital. In each loan contract, if the borrower's draw is above a cutoff value of the idiosyncratic shock, denoted as  $\bar{\omega}^i$ , he makes a fixed payment and the lender does not monitor; otherwise, the borrower defaults, the lender monitors and takes whatever remains. (Townsend, 1979)

A financial contract decides both the size of credit and the split of the revenue between borrowers and lenders. Suppose the idiosyncratic shock has a cumulative distribution function  $F^i(\omega^i)$ . The share of revenue that goes to the lender before monitoring,  $\Gamma^i(\bar{\omega}^i)$ , is given by

$$\Gamma^i(\bar{\omega}^i) \equiv G^i(\bar{\omega}^i) + [1 - F^i(\bar{\omega}^i)]\bar{\omega}^i,$$

where  $G^i(\bar{\omega}^i) \equiv \int_0^{\bar{\omega}^i} \omega^i dF^i(\omega^i)$  accounts for what the lender gets on average (before monitoring occurs) if the borrower defaults. Also define  $\Psi^i(\bar{\omega}^i) \equiv \Gamma^i(\bar{\omega}^i) - \mu^i G^i(\bar{\omega}^i)$  as the fraction of revenue that goes to the lender after monitoring.

The chained credit contract is as follows. Given the banks' net worth  $N^F$  and the entrepreneurs' net worth  $N^E$ , the banks maximise their expected profits subject to the participation constraints of the entrepreneurs and the investors.<sup>1</sup> Banks choose the capital  $K$ , and the cutoff values  $\bar{\omega}^E, \bar{\omega}^F$  to maximise their profit,

$$(1 - \Gamma^F(\bar{\omega}^F))R^F(K - N^E), \quad (1)$$

subject to

$$R^F(K - N^E) = \Psi^E(\bar{\omega}^E)R^E K, \quad (2)$$

$$(1 - \Gamma^E(\bar{\omega}^E))R^E K \geq R^E N^E, \quad (3)$$

$$\Psi^F(\bar{\omega}^F)R^F(K - N^E) = R(K - N^E - N^F). \quad (4)$$

The objective, Equation (1), is the fraction  $(1 - \Gamma^F(\bar{\omega}^F))$  of bank revenue  $R^F(K - N^E)$  retained by banks. Equation (2) says the banks take a fraction  $\Psi^E(\bar{\omega}^E)$  of the entrepreneurs' revenue. Equation (3) is the participation constraint of the entrepreneurs which requires that the revenue retained by the entrepreneurs has to be greater than the profit if the entrepreneurs operate only with their own fund,  $R^E N^E$ . Equation (4) is the participation constraint of the investors. It requires that the fraction of bank revenue that goes to the investors, after deducting monitoring cost, has to be big enough to pay for the risk-free return  $R$  of the investors' deposits.

We can solve the bank profit maximisation problem to get:<sup>2</sup>

$$\frac{R^E}{R} = \rho^E(\bar{\omega}^E) \times \rho^F(\bar{\omega}^F) \quad (5)$$

where

$$\rho^i(\bar{\omega}^i) \equiv \frac{(\Gamma_\omega^i(\bar{\omega}^i)/\Psi_\omega^i(\bar{\omega}^i))}{(1 - \Gamma^i(\bar{\omega}^i)) + \Psi^i(\bar{\omega}^i)(\Gamma_\omega^i(\bar{\omega}^i)/\Psi_\omega^i(\bar{\omega}^i))} \geq 1, \quad \frac{d\rho^i(\bar{\omega}^i)}{d\bar{\omega}^i} > 0.$$

The optimal choice by the banks satisfies Equation (5), which is the key result of this paper. It says that the external finance premium,  $R^E/R$ , can be factored into two components. One component is a function of the cutoff value in the loan contract between

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<sup>1</sup>Given the linear monitoring technology, in the optimal contract, all banks choose the same capital to net worth ratios so that aggregation is trivial. For simplicity, we use aggregate variables in this paper.

<sup>2</sup>The derivation is provided in the Appendix.

the banks and the entrepreneurs,  $\rho^E(\bar{\omega}^E)$ , and the other component is a function of the cutoff value in the loan contract between the banks and the investors,  $\rho^F(\bar{\omega}^F)$ . Each of these two components has an identical structure with the external financial premium in the original BGG contract.<sup>3</sup>

The function  $\rho^i(\bar{\omega}^i)$  is bounded below by unity and is increasing in  $\bar{\omega}^i$  because monitoring is costly.<sup>4</sup> Equation (5) shows that the external finance premium is increasing in both  $\bar{\omega}^E$  and  $\bar{\omega}^F$ . This is because resources are wasted in monitoring in each of the two loan contracts.

Ultimately, the external finance premium  $R^E/R$  is determined by the capital to net worth ratios of the banks and the entrepreneurs. Differentiating the participation constraint of the entrepreneurs, Equation (3), we obtain:

$$\frac{d\bar{\omega}^E}{d(K/N^E)} = \frac{1 - \Gamma^E}{\Gamma_\omega^E} \times \frac{1}{(K/N^E)} > 0, \quad (6)$$

since  $\Gamma_\omega^i(\bar{\omega}^i) = 1 - F^i(\bar{\omega}^i) > 0$ . In addition, keeping the entrepreneurs' capital to net worth ratio unchanged, a rise in the bank's capital to net worth ratio increases  $\bar{\omega}^F$ . Substituting Equations (2) and (5) into the participation constraint of the investors, Equation (4), and differentiating, we obtain:

$$\frac{d\bar{\omega}^F}{d(K/N^F)} = \left( \frac{\Psi_\omega^F}{\Psi^F} + \frac{\rho_\omega^F}{\rho^F} \right)^{-1} \frac{K}{K - N^E - N^F} (N^F/K)^2 > 0, \quad (7)$$

since  $\Psi_\omega^i > 0$  in the optimal contract.<sup>5</sup> These mean that  $R^E/R$  is increasing in the capital

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<sup>3</sup>In the BGG model, there are no banks and investors lend directly to entrepreneurs. In their model, the first order condition is  $R^E/R = \rho^E(\bar{\omega}^E)$ . In fact, the BGG contract is a special case of the chained credit contract when investors can monitor banks costlessly, i.e.  $\mu^F = 0$ .

<sup>4</sup>The appendix of BGG presents a formal proof.

<sup>5</sup>The proof is as follows:

$$\begin{aligned} G_\omega^i(\bar{\omega}^i) &= \bar{\omega}^i f^i(\bar{\omega}^i) = \bar{\omega}^i h^i(\bar{\omega}^i)(1 - F^i(\bar{\omega}^i)) > 0, \\ \Gamma_\omega^i(\bar{\omega}^i) &= G_\omega^i(\bar{\omega}^i) + (1 - F^i(\bar{\omega}^i)) - \bar{\omega}^i f^i(\bar{\omega}^i) = 1 - F^i(\bar{\omega}^i) > 0, \\ \Psi_\omega^i(\bar{\omega}^i) &= \Gamma_\omega^i(\bar{\omega}^i) - \mu^i G_\omega^i(\bar{\omega}^i) = (1 - F^i(\bar{\omega}^i))(1 - \mu^i \bar{\omega}^i h^i(\bar{\omega}^i)), \end{aligned}$$

where  $h^i(\bar{\omega}^i) = f^i(\bar{\omega}^i)/(1 - F^i(\bar{\omega}^i))$  is the hazard rate, and for  $i \in \{E, F\}$ . For the log-normal distribution,  $\bar{\omega}^i h^i(\bar{\omega}^i) = 0$  when  $\bar{\omega}^i = 0$ ,  $\lim_{\bar{\omega}^i \rightarrow \infty} \bar{\omega}^i h^i(\bar{\omega}^i) = \infty$ , and  $\bar{\omega}^i h^i(\bar{\omega}^i)$  is increasing in  $\bar{\omega}^i$ . Hence, there exists an  $\bar{\omega}^{i*}$  such that  $\Psi_\omega^i(\bar{\omega}^i) > 0$  for  $\bar{\omega}^i < \bar{\omega}^{i*}$  and  $\Psi_\omega^i(\bar{\omega}^i) < 0$  for  $\bar{\omega}^i > \bar{\omega}^{i*}$ . For any  $\bar{\omega}_1^i$  such that  $\bar{\omega}_1^i > \bar{\omega}^{i*}$ , there exist a  $\bar{\omega}_2^i$  such that  $\bar{\omega}_2^i < \bar{\omega}^{i*} < \bar{\omega}_1^i$  and  $\Psi^i(\bar{\omega}_2^i) = \Psi^i(\bar{\omega}_1^i)$ . Since the smaller  $\bar{\omega}_2^i$  implies a smaller bankruptcy rate for the borrower than  $\bar{\omega}_1^i$  while keeping the lender's share of profit unchanged, any  $\bar{\omega}_1^i > \bar{\omega}^{i*}$  will never be chosen. Hence,  $\bar{\omega}^i$  has an interior solution and in the optimal contract  $\Psi_\omega^i(\bar{\omega}^i) > 0$ .

to net worth ratio of the banks and the entrepreneurs.

### 3 Global Banks

This section applies the chained credit contract to global banks. Our analytical solution shows that global banks give rise to positive comovement in external finance premia across countries.

Global banks take deposits from home and foreign investors and lend to home and foreign entrepreneurs. Home and foreign investors face the common risk-free interest rate,  $R$ . However, the cost of funds for home and foreign entrepreneurs may be different. Global banks maximise their expected profits subject to the participation constraints of the home and foreign entrepreneurs and of the investors. Global banks choose  $\{K, K^*, \bar{\omega}^E, \bar{\omega}^{E*}, \bar{\omega}^F\}$  to maximise their profit (We use a superscript  $*$  to denote a foreign variable.):

$$(1 - \Gamma^F(\bar{\omega}^F))[R^F(K - N^E) + R^{F*}(K^* - N^{E*})], \quad (8)$$

subject to

$$R^F(K - N^E) = \Psi^E(\bar{\omega}^E)R^E K, \quad (9)$$

$$R^{F*}(K^* - N^{E*}) = \Psi^{E*}(\bar{\omega}^{E*})R^{E*} K^*, \quad (10)$$

$$(1 - \Gamma^E(\bar{\omega}^E))R^E K \geq R^E N^E, \quad (11)$$

$$(1 - \Gamma^{E*}(\bar{\omega}^{E*}))R^{E*} K^* \geq R^{E*} N^{E*}, \quad (12)$$

$$\Psi^F(\bar{\omega}^F)[R^F(K - N^E) + R^{F*}(K^* - N^{E*})] = R(K + K^* - N^E - N^{E*} - N^F). \quad (13)$$

Global banks receive revenue from their lending to domestic entrepreneurs  $R^F(K - N^E)$  and foreign entrepreneurs  $R^{F*}(K^* - N^{E*})$ . They pool their revenues and retain a fraction  $(1 - \Gamma^F(\bar{\omega}^F))$  as profits. Equations (9) and (10) define the revenues from lending to home and foreign entrepreneurs. Equations (11) and (12) are the participation constraints of the entrepreneurs, analogous to Equation (3). Equation (13) is the participation constraint of the home and foreign investors, analogous to Equation (4).

A similar derivation as above would result in the following optimal conditions for the

global bank:

$$\frac{R^E}{R} = \rho^E(\bar{\omega}^E) \times \rho^F(\bar{\omega}^F), \quad (14)$$

$$\frac{R^{E*}}{R} = \rho^{E*}(\bar{\omega}^{E*}) \times \rho^F(\bar{\omega}^F). \quad (15)$$

The external finance premia for home entrepreneurs and foreign entrepreneurs have a common component,  $\rho^F(\bar{\omega}^F)$ . This is because the global banks are subject to a common participation constraint with both groups of investors. This common component induces positive comovement of home and foreign external finance premia.

Suppose home entrepreneurs demand an additional unit of capital  $K$ , for given  $K^*$ ,  $N^E$ ,  $N^{E*}$  and  $N^F$ , what will happen to the domestic and foreign external finance premia?

Figure 1 provides a numerical example of this comparative statics exercise.<sup>6</sup> We assume the home and foreign economies are identical and follow the assumptions from HSU (2010) in our calibration. Specifically, we assume (1)  $F^F(\bar{\omega}^F) = 0.02$ ; (2)  $F^E(\bar{\omega}^E) = 0.02$ ; (3)  $K/N^F = 5$ ; (4)  $K/N^E = 2$ ; (5)  $Z^E/Z^F = 1.023^{1/4}$ , where  $Z^E, Z^F$  are the contractual interest rates for the two loan contracts; (6)  $R^E/R = 1.02^{1/4}$ . Given these assumptions, the credit parameters are found to be  $\bar{\omega}^E = \bar{\omega}^{E*} = 0.501, \bar{\omega}^F = 0.797, \sigma^E = \sigma^{E*} = 0.313, \sigma^F = 0.107, \mu^E = \mu^{E*} = 0.0187, \mu^F = 0.0303$ .<sup>7</sup> In the comparative statics exercise, we change  $K$  in the home country exogenously and solve for  $\{\bar{\omega}^E, \bar{\omega}^{E*}, \bar{\omega}^F\}$ , fixing  $K^*, N^E, N^{E*}, N^F$  and keeping the credit parameters  $\{\sigma^E, \sigma^{E*}, \sigma^F, \mu^E, \mu^{E*}, \mu^F\}$  constant.

[Figure 1 here]

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<sup>6</sup>Mathematically, we can show that:

$$\begin{aligned} \frac{d\bar{\omega}^E}{d\ln(K)} &= \frac{(1 - \Gamma^E)}{\Gamma_\omega^E} > 0, \\ \frac{d\bar{\omega}^F}{d\ln(K)} &= \frac{1 - \Psi^F \Psi^E \frac{R^E}{R} \left(1 + \left(\frac{\Psi_\omega^E}{\Psi_\omega^{E*}} + \frac{\rho_\omega^E}{\rho_\omega^{E*}}\right) \frac{(1 - \Gamma^E)}{\Gamma_\omega^E}\right)}{\frac{\Psi_\omega^F}{\Psi_\omega^{F*}} \left(\frac{K + K^* - N^E - N^{E*} - N^F}{K}\right) + \Psi^F \Psi^E \frac{R^E}{R} \frac{\rho_\omega^F}{\rho_\omega^{F*}}}. \end{aligned}$$

<sup>7</sup>The contractual interest rates  $Z^E, Z^F$  are defined as follows:

$$\begin{aligned} \bar{\omega}^E R^E K &= Z^E (K - N^E), \\ \bar{\omega}^F [R^F (K - N^E) + R^{F*} (K^* - N^{E*})] &= Z^F (K + K^* - N^E - N^{E*} - N^F). \end{aligned}$$

There is a one-to-one correspondence between  $\bar{\omega}^E$  and  $Z^E$ , and between  $\bar{\omega}^F$  and  $Z^F$ .

The left panel of Figure 1 shows the home and foreign external finance premia against the capital to net worth ratio of home entrepreneurs. The external finance premia in the two countries positively comove. The external finance premium in the home economy increases more, as it is driven by a rise in both  $\bar{\omega}^E$  and  $\bar{\omega}^F$ . The foreign external finance premium rises only as a result of a rise in  $\bar{\omega}^F$ . Hence, the dashed line also shows the contribution of  $\rho^F(\bar{\omega}^F)$  to the home external finance premium, or the ‘common lender’ effect. With this calibration, more than half of the home external finance premium can be accounted for by the rise in  $\bar{\omega}^F$ . The right panel shows the cutoff values against the capital to net worth ratio of home entrepreneurs. Clearly,  $\bar{\omega}^E$  moves by more than  $\bar{\omega}^F$ . But since banks have higher leverage than the entrepreneurs,  $d\rho^F(\bar{\omega}^F)/\bar{\omega}^F > d\rho^E(\bar{\omega}^E)/\bar{\omega}^E$ , and therefore  $\rho^F(\bar{\omega}^F)$  increases markedly even though the rise in  $\bar{\omega}^F$  itself is small.<sup>8</sup> This suggests that the role of high bank leverage in increasing volatility.

## 4 Conclusion

We showed that the solution of the chained credit contract has a simple, symmetric structure analogous to the one in BGG. We applied the chained credit contract to analyse a global bank. The simple structure of the optimality conditions allowed us to identify the common lender effect which drives the positive comovement of the external finance premia across economies.

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<sup>8</sup>With our calibrations,  $d\bar{\omega}^E/d\ln(K) = 0.51$ ,  $d\bar{\omega}^F/d\ln(K) = 0.20$ ,  $\rho_\omega^E = 0.02$  and  $\rho_\omega^F = 0.07$ .



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## A Derivation of Equation (5)

We first use Equation (2) to replace  $R^F(K - N^E)$  in the objective (1) and the household participation constraint (4). We write down the Lagrangian with  $\lambda_1$  and  $\lambda_2$  denoting the Lagrangian multipliers associated with the entrepreneurs' participation constraint and the investors' participation constraint as follows:

$$L = (1 - \Gamma^F(\bar{\omega}^F))\Psi^E(\bar{\omega}^E)R^E K + \lambda_1[\Psi^F(\bar{\omega}^F)\Psi^E(\bar{\omega}^E)R^E K - R(K - N^E - N^F)] \\ + \lambda_2[(1 - \Gamma^E(\bar{\omega}^E))K - N^E]$$

The first order conditions are:

$$\begin{aligned} K &: (1 - \Gamma^F)\Psi^E R^E + \lambda_1[\Psi^F \Psi^E R^E - R] + \lambda_2(1 - \Gamma^E) = 0 \\ \bar{\omega}^F &: -\Gamma_\omega^F + \lambda_1 \Psi_\omega^F = 0 \\ \bar{\omega}^E &: (1 - \Gamma^F)\Psi_\omega^E R^E + \lambda_1 \Psi^F \Psi_\omega^E R^E - \lambda_2 \Gamma_\omega^E = 0 \end{aligned}$$

Rearranging the last two equations, we solve for the Lagrange multipliers as follows:

$$\begin{aligned} \lambda_1 &= \frac{\Gamma_\omega^F}{\Psi_\omega^F}, \\ \lambda_2 &= \left( \frac{(1 - \Gamma^F)\Psi_\omega^E}{\Gamma_\omega^E} + \frac{\Gamma_\omega^F \Psi^F \Psi_\omega^E}{\Gamma_\omega^E \Psi_\omega^F} \right) R^E. \end{aligned}$$

These are put into the first order condition for  $K$ :

$$\begin{aligned} 0 &= (1 - \Gamma^F)\Psi^E R^E + \frac{\Gamma_\omega^F}{\Psi_\omega^F}[\Psi^F \Psi^E R^E - R] + (1 - \Gamma^E) \left( \frac{(1 - \Gamma^F)\Psi_\omega^E}{\Gamma_\omega^E} + \frac{\Gamma_\omega^F \Psi^F \Psi_\omega^E}{\Gamma_\omega^E \Psi_\omega^F} \right) R^E \\ \frac{R^E}{R} &= \frac{\Gamma_\omega^E \Gamma_\omega^F}{\Psi^E \Gamma_\omega^E \Psi^F \Gamma_\omega^F + \Psi^E \Gamma_\omega^E (1 - \Gamma^F) \Psi_\omega^F + (1 - \Gamma^E) \Psi_\omega^E \Psi^F \Gamma_\omega^F + (1 - \Gamma^E) \Psi_\omega^E (1 - \Gamma^F) \Psi_\omega^F} \\ &= \left( \frac{(\Gamma_\omega^E / \Psi_\omega^E)}{(1 - \Gamma^E) + \Psi^E (\Gamma_\omega^E / \Psi_\omega^E)} \right) \left( \frac{(\Gamma_\omega^F / \Psi_\omega^F)}{(1 - \Gamma^F) + \Psi^F (\Gamma_\omega^F / \Psi_\omega^F)} \right) \\ &= \rho^E \times \rho^F \end{aligned}$$

Figure 1. (2-column fitting image)  
Caption: Comparative Statics: Vary borrowing of home entrepreneurs.