Supplementary Information for "Negative optical torque"

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The role of symmetry, compactness, and absorption in optical torque

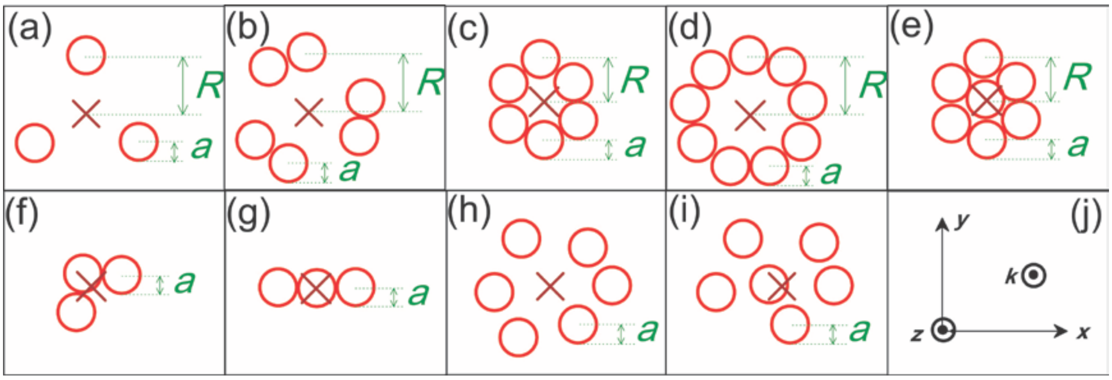
To illustrate explicitly that compact structures tend to experience larger positive optical torque (POT), while discrete rotational symmetry favors negative optical torque (NOT), we plotted the following quantities in Fig. S2 for the structures shown in Fig. S1,

$$
\begin{aligned}
I_{-} &= \frac{1}{N_{\mathrm{sphere}} R} \int_{k_1 a}^{k_2 a} \Gamma_z \Theta(-\Gamma_z) \mathrm{d}(ka) \\
I_{+} &= \frac{1}{N_{\mathrm{sphere}} R} \int_{k_1 a}^{k_2 a} \Gamma_z \Theta(\Gamma_z) \mathrm{d}(ka)
\end{aligned} \tag{S1}
$$

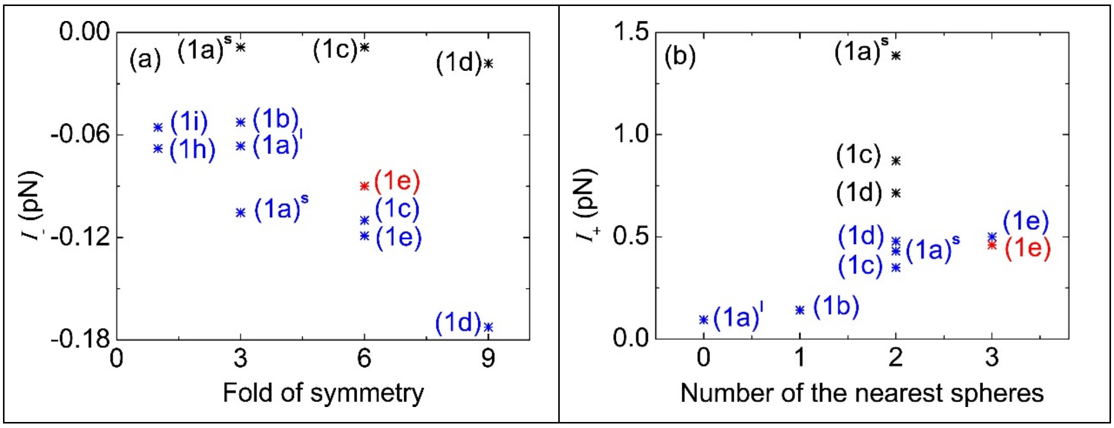
where k_1a =2.1 and k_2a =10.2 are arbitrarily chosen, N_{sphere} is the number of spheres in the structure, $\Theta(x)$ is the Heaviside step function, and the incident wave is a left circularly polarized plane wave with an intensity of $1mW/\mu m^2$. In simple words, I_+ and I_- are obtained by integrating only POT and NOT, respectively. Accordingly, they characterize the strength of POT and NOT over the spectrum domain. As seen from the blue data points (corresponding to lossless dielectric structures) in Fig. S2(a), I_- increases in magnitude with degree of discrete rotational symmetry, indicating discrete rotational symmetry favors the realization of NOT. Similarly, from Fig. S2(b), I_+ increases with the number of nearest neighbor, indicating that compactness enhances POT. We note that I_+ and I_- are of comparable magnitude, therefore NOT is neither small nor exceptional, albeit less likely to be observed in comparison with POT. Furthermore, I_+ and I_- are normalized by sphere number N_{sphere} and structural size R. Consequently the observed effect is a collective effect, which cannot be explained solely by the superposition of spheres or the overall structural size. A typical example is shown in Fig. S3 to demonstrate more explicitly the role of symmetry and compactness. In Fig. S3 where the structures are of approximately the same size, by comparing the red and blue lines, it can be seen that as the degree of rotational symmetry increases, NOT is enhanced. Similarly, by comparing blue and black lines, the more compact black line has considerably stronger POT.

One may anticipate that NOT is vulnerable to material absorption and occurs exclusively for non-absorptive particles, as every photon absorbed will impart an angular momentum of $m_i\hbar$ to the structure. While absorption does introduce a positive bias, as shown in Fig. S2, where the black points (with absorption) are shifted toward the positive side compared to their corresponding blue points (without absorption).

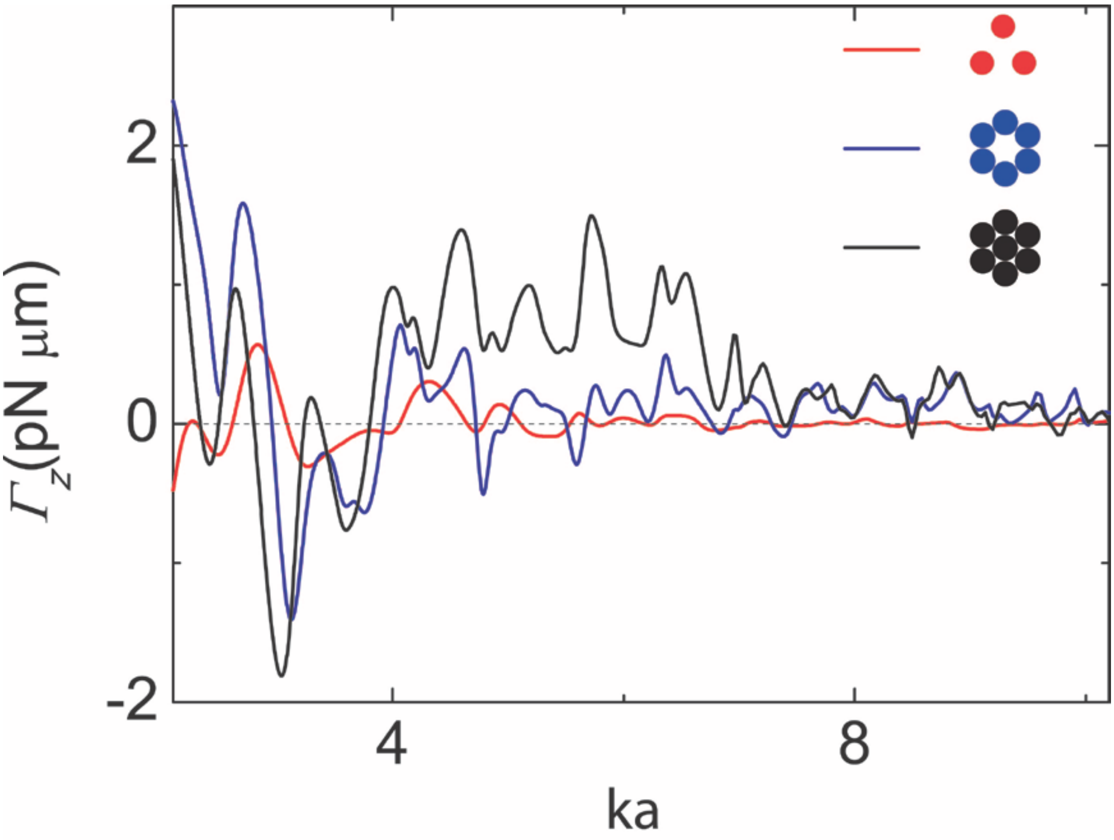
However, NOT survives for those structures that have sufficiently high discrete rotational symmetry [see, Fig. S2(a)]. A more explicit example is shown in Fig. S4. As a direct consequence of absorption, the torque on the three-fold symmetric structure is still oscillatory but biased to the positive side. By progressively increases the rotational symmetry from three to six and to nine-fold, the oscillation is significantly enhanced compared to the bias, therefore NOT becomes significant. These indicate that discrete rotational symmetry enhances NOT even in the presence of material absorption.



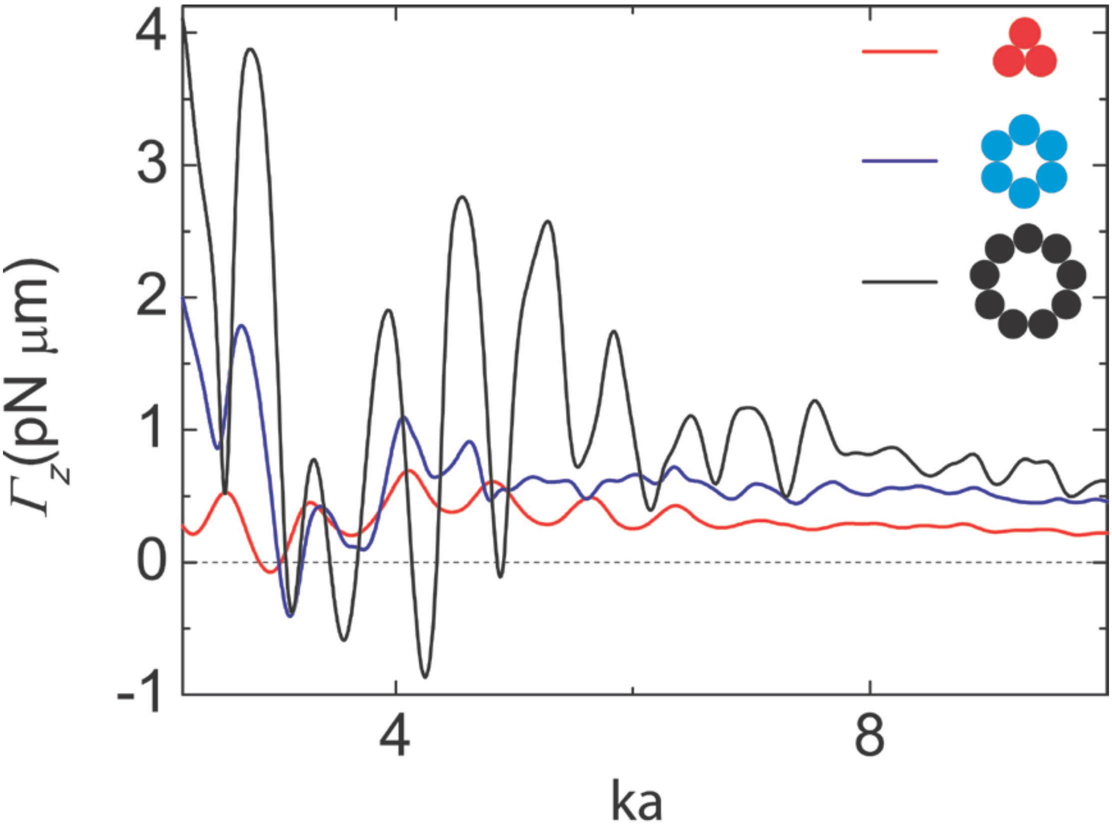
Supplementary Figure S1 | Examples of planar structures that can realize NOT. (a)-(i) structures composed of identical dielectric spheres. Here, a is the spheres' radius and R is the average distance between the spheres' centers and the origin. The cross marks the center of mass, which coincides with the axis of rotation. Panel (j) shows the coordinate system. The incident plane wave has a k -vector (k) along the z -axis.



Supplementary Figure S2 | (a) I_{-}**, which indicates the strength of NOT, versus degree of rotational symmetry (b)** I_{+}**, which indicates the strength of POT, versus number of the nearest spheres for various structures composed of dielectric spheres (** $a = 0.49\mu m$ **).** Blue: lossless structures with $\varepsilon_r = 2.4649$. Black: absorptive structures with $\varepsilon_r = 2.4649 + 0.05i$. Red: Structure of Fig. 1(e) with the central sphere having $\varepsilon_r = 1.44$, while all the other spheres having $\varepsilon_r = 2.4649$. Each structure is defined in Fig. S1. (1a)^s: Fig. S1(a) with $R = 0.58\mu m$, (1a)^l: Fig. S1(a) with $R = 1.46\mu m$, (1b): Fig. S1(b) with $R = 1.46\mu m$, (1c): Fig. S1(c) with $R = 1.0\mu m$, (1d): Fig. S1(d) with $R = 1.46\mu m$, (1e): Fig. S1(e) with $R = 1.0\mu m$, (1h): Fig. S1(h) with $R = 1.3\mu m$, (1i): Fig. S1(i) with $R = 1.1\mu m$.



Supplementary Figure S3 | Optical torque versus *ka* **for different structures composed of identical dielectric spheres (**$\varepsilon_r = 2.4649$**) in air.** These three structures all have the same radius, i.e. the separation between the sphere and structural center is the same. Here, k is the wave number in the surrounding medium, and a is the radius of the identical spheres comprising the structures. The degree of discrete rotational symmetry enhances NOT, while the compactness (as measured by the number of nearest neighbors) enhances POT.



Supplementary Figure S4 | Optical torque versus *ka* for different structures composed of absorptive spheres ($\varepsilon_r = 2.4649 + 0.05i$) in air. Higher degree of discrete rotational symmetry is seen to considerably enhance NOT against material absorption.

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