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The first exit time and ruin time for a risk process with reserve-dependent income

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Abstract

This paper investigates the first exit time and the ruin time of a risk reserve process with reserve-dependent income under the assumption that the claims arrive as a Poisson process. We show that the Laplace transform of the distribution of the first exit time from an interval satisfies an integro-differential equation. The exact solution for the classical model and for the Embrechts-Schmidli model are derived.

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1. Introduction

Consider the risk reserve process $(X_t)_{t\geq 0}$ described by

$$X_t = x + \int_0^t c(X_s)\mathrm{d}s - S_t, \quad t \geq 0,$$

where x is the non-negative initial capital, $c(\cdot)$ a continuously differentiable Lipschitz function which represents the positive reserve-dependent income rate, and $(S_t)_{t\geq 0}$ the aggregate claim process defined by $S_t = \sum_{i=1}^{N_t} Y_i$ such that $(N_t)_{t\geq 0}$ is a Poisson process with intensity λ and $(Y_i)_{i\geq 1}$ a sequence of independent claim amounts. Assume that $(N_t)_{t\geq 0}$ and Y_i's are independent and Y_i's are identically distributed having an absolutely continuous distribution function P with $P(0) = 0$ and a finite mean. An important particular case is the classical risk process obtained by taking $c(\cdot)$ as a constant.

From Dassios and Embrechts (1989) or Embrechts and Schmidli (1994) we know that $(X_t)_{t\geq 0}$ is a piecewise deterministic Markov process (PDMP) taking values in $\mathbb{R}$ with extended generator $\mathcal{A}$ that satisfies

$$\mathcal{A}f(x) = \chi f(x) + \lambda \int_0^\infty (f(x-y) - f(x))\mathrm{d}P(y),$$

where f belongs to the domain $\mathcal{D}(\mathcal{A})$ of the generator $\mathcal{A}$ of $(X_t)_{t\geq 0}$, $\chi = c(x)\frac{\mathrm{d}}{\mathrm{d}x}$ is the vector field of the integral curves of the PDMP.

Assume that all processes and random variables are defined on a filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F})_{t\geq 0})$, where $\mathcal{F}_t$ is right-continuous. Denoted by $\mathbf{P}_x$ and $\mathbf{E}_x$ the conditional distribution of X_t and its expectation operator, given that $X_0 = x$. For two constants $b < a$, define the first exit time from (b, a) by $\tau_{a,b} = \inf\{t > 0 : X_t \leq b \text{ or } X_t \geq a\}$ and so $\tau_{\infty,0}$ is the time of ruin. Note that $\tau_{a,b}$ is a stopping time with respect to $(\mathcal{F}_t)_{t\geq 0}$. In order to avoid $\mathbf{P}_x(\tau_{\infty,0} < \infty) = 1$, we assume the net profit condition $\mathbf{E}_0(X_t) > 0$ for $t \geq 0$.

We conclude this section by showing that the Laplace transform of $\tau_{a,b}$ satisfies the integro-differential equation given in (1.1).

Lemma 1.1. *For $\alpha > 0$, consider the Laplace transform $V(x) := \mathbf{E}_x e^{-\alpha\tau_{a,b}}, b < x < a$. Then V is the unique solution in $\mathcal{D}(\mathcal{A})$ of*

$$\chi V(x) - \lambda \int_b^x V(t) \mathrm{d}P(x-t) + \lambda(1 - P(x-b)) = (\lambda + \alpha)V(x), \ x \in (b, a), \tag{1.1}$$

with the boundary condition $V(a) = 1$.

Proof. The Markov property implies that for $s < \tau_{a,b}$

$$\mathbf{E}_x \left(V(X_{\tau_{a,b}}) e^{-\alpha\tau_{a,b}} | \mathcal{F}_s \right) = V(X_s) e^{-\alpha s},$$

where $\mathcal{F}_s$ is the σ-algebra generated by $\{X_t, t \le s\}$. The left-hand side is a local martingale on $[0, \tau_{a,b})$, and so is $e^{-\alpha s} V(X_s)$. Note that

$$V(x) = 1 - \alpha \mathbf{E}_x \int_0^\infty e^{-\alpha t} \ \mathbf{1}(\tau_{a,b} > t) \mathrm{d}t.$$

By using the same method as in the proof of the first part of Theorem 32.2 in Davis (1993), we have $V \in \mathcal{D}(\mathcal{A})$. Applying the PDMP Differential Formula (Davis, 1993, Theorem 31.3) for $s \in [0, \tau_{a,b})$ leads to

$$V(X_s) e^{-\alpha s} - V(x) = \int_0^s e^{-\alpha t} (\mathcal{A} - \alpha) V(X_t) \mathrm{d}t + \text{a local martingale.} \tag{1.2}$$

It follows that the first integral on the right-hand side of (1.2) is a local martingale. Hence, the smoothness of V implies $\mathcal{A}V(x) - \alpha V(x) = 0$ in (b, a). Since $V(x) = 1$ for $x < b$, equation (1.1) follows. The boundary condition $V(a) = 1$ is obvious.

To show the uniqueness, suppose V is a function that satisfies (1.1), and so $\mathcal{A}V(x) = \alpha V(x)$. We now prove that $V(x) = \mathbf{E}_x e^{-\alpha\tau_{a,b}}$. If $V \in \mathcal{D}(\mathcal{A})$, then $V e^{-\alpha s} \in \mathcal{D}(\mathcal{A})$. Applying the PDMP Differential Formula for $s \in [0, \tau_{a,b})$ leads to (1.2) again. Now in view of (1.1), we observe that $M_s := V(X_s) e^{-\alpha s}$ is a local martingale. Since V is bounded, M_s is a bounded martingale. Note that $\tau_{a,b}$ is a stopping time and $\mathbf{E}_x \tau_{a,b} < \infty$. Thus, by the Optional-Stopping Theorem and the dominated convergence theorem, the result follows.

Remark 1.1. Equation (1.1) remains true when $b \to -\infty$ or $a \to \infty$, but the boundary condition becomes $\lim_{a\to\infty} V(a) = 0$.

2. Application to the classical model

We apply the results in the last section to the classical model, i.e. $c(x) = c$, a constant. The Laplace transform of a function will be denoted by putting a hat on the function. For example, $\hat{g}(\beta) = \int_0^\infty \mathrm{e}^{-\beta x} g(x)\mathrm{d}x$.

Assumption 2.1. We assume that there exists a $\beta_\infty \in [-\infty, 0)$ such that $\hat{p}(\beta) \to \infty$ when $\beta \downarrow \beta_\infty$.

The following theorem is a classical result, see Asmussen (2000, p. 109, Corollary 3.5) for a proof using renewal theory.

Theorem 2.1. *Let* $h(x) = \mathbf{E}_x e^{-\alpha\tau_{\infty,0}}$ *for* $x > 0$. *Its Laplace transform is given by*

$$\hat{h}(\beta) = \begin{cases} \dfrac{\frac{\lambda}{\beta}\hat{p}(\beta) - \frac{\lambda}{\beta} + ch(0)}{\lambda\hat{p}(\beta) + \beta c - \lambda - \alpha}, & \hat{p}(\beta) < \infty, \\ \dfrac{1}{\beta}, & \hat{p}(\beta) = \infty. \end{cases} \tag{2.1}$$

Proof. By Lemma 1.1, the function $h(x)$ is the unique solution in $\mathcal{D}(\mathcal{A})$ of the equation

$$ch'(x) - \lambda \int_0^x h(t)\mathrm{d}P(x-t) + \lambda(1 - P(x)) = (\lambda + \alpha)h(x), \ x \in (0, \infty), \tag{2.2}$$

with the boundary condition $h(\infty) = 0$. If $\hat{p}(\beta) < \infty$, equation (2.1) follows from taking the Laplace transforms of the both sides of (2.2). If $\hat{p}(\beta) = \infty$, $\hat{h}(\beta)$ takes the limiting value as $\hat{p}(\beta) \to \infty$.

Before discussing the inversion formula of (2.1), we need the following lemma.

Lemma 2.1. *For positive* λ *and* c *and nonnegative* α, *consider the function* $k_\alpha(\beta) = \lambda\hat{p}(\beta) + \beta c - \lambda - \alpha$ *on the complex plane, where* $\hat{p}$ *is the Laplace transform of* p.

(1) If $\alpha > 0$, then $k_\alpha(\cdot)$ has a unique positive zero and a negative zero; the remaining zeros are either negative or having negative real parts.

(2) If $\alpha = 0$, then $k_\alpha(\cdot)$ has at least two real zeros, one is a negative and the other is $\beta = 0$; the remaining zeros are either negative or having negative real parts.

Proof. If $\beta \in \mathbb{R}$, $k_\alpha(\beta)$ is a continuous convex function that tends to infinity as $\beta \downarrow \beta_\infty$ or $\beta \uparrow \infty$ and takes the value $-\alpha$ at $\beta = 0$. Hence, if $\alpha > 0$, $k_\alpha(\cdot)$ has a unique positive zero and a negative zero. Moreover, since the positive zero is an increasing function of α, it becomes 0 when $\alpha = 0$. By the Argument Principle (e.g. Ablowits and Fokas, 1997, p. 259), it can be proved that the remaining zeros are all located on the negative half-plane.

For simplicity, we assume that all zeros $\beta_1(\alpha), \beta_2(\alpha), \ldots$ of $k_\alpha(\cdot)$ are simple and that $\mathrm{Re}(\beta_j(\alpha)) < 0$ for $j \geq 2$ and $\beta_1(\alpha) > 0$ for $\alpha > 0$ and $\beta_1(0) = 0$.

Theorem 2.2. *If the function $\hat{h}(\cdot)$ in (2.1) is analytic on the complex plane except for $\beta_1(\alpha), \beta_2(\alpha), \ldots$, then for $\alpha > 0$,*

$$\mathbf{E}_x e^{-\alpha\tau_{\infty,0}} = \begin{cases} \sum_{j\geq 2} \dfrac{\frac{\lambda}{\beta_j(\alpha)}\left(\hat{p}(\beta_j(\alpha)) - 1\right) - \frac{\lambda}{\beta_1(\alpha)}\left(\hat{p}(\beta_1(\alpha)) - 1\right)}{\lambda\hat{p}'(\beta_j(\alpha)) + c} e^{\beta_j(\alpha)x}, & x > 0, \\ 1 - \dfrac{\alpha}{c\beta_1(\alpha)}, & x = 0, \end{cases} \tag{2.3}$$

and

$$\mathbf{P}_x(\tau_{\infty,0} < \infty) = \sum_{j\geq 2} \frac{\frac{\lambda}{\beta_j(0)}\left(\hat{p}(\beta_j(0)) - 1\right) + \lambda EY_1}{\lambda\hat{p}'(\beta_j(0)) + c} e^{\beta_j(0)x}, \quad x > 0. \tag{2.4}$$

Proof. The inversion formula is

$$h(x) = \frac{1}{2\pi i} \lim_{w\to\infty} \int_{\mu-iw}^{\mu+iw} \hat{h}(\beta)\mathrm{e}^{\beta x}\mathrm{d}\beta,$$

where $\mu\,(> \beta_1(\alpha))$ is a constant. Since $\beta\hat{h}(\beta)\mathrm{e}^{\beta x} \to 0$ as $|\beta| \to \infty$, it follows from Cauchy's Residue Theorem and Jordan's Lemma that

$$h(x) = \sum_{j\geq 1} \mathrm{Res}(\hat{h}(\beta_j(\alpha))\mathrm{e}^{\beta_j(\alpha)x}) = \sum_{j\geq 1} \frac{\frac{\lambda}{\beta_j(\alpha)}\left(\hat{p}(\beta_j(\alpha)) - 1\right) + ch(0)}{\lambda\hat{p}'(\beta_j(\alpha)) + c} \mathrm{e}^{\beta_j(\alpha)x}.$$

The boundedness of $h(\cdot)$ implies

$$\frac{\lambda}{\beta_1(\alpha)}\left(\hat{p}(\beta_1(\alpha))-1\right)+ch(0)=0,$$

as $\beta_1(\alpha)>0$. Equation (2.3) follows. By letting $\alpha\to 0$, we get (2.4).

Remark 2.1. In particular, when $x=0$, (2.3) coincides with Asmussen (2000, p. 109, Corollary 3.4). From (2.4) we can get the well-known Cramér-Lundberg approximation (Rolski *et al.*, 1999, p. 172).

3. Application to the Embrechts-Schmidli model

In this section, instead of the classical model, we consider that a company earns interest at an interest rate $\rho_1>0$ when the reserve is positive, and borrows money at an interest rate $\rho_2>0$ when the reserve is negative. This model was considered by Embrechts and Schmidli (1994). The vector field of the integral curves in the model is

$$\chi:=\begin{cases}(c+\rho_1 x)\dfrac{\partial}{\partial x}, & x\geq 0,\\[2mm] (c+\rho_2 x)\dfrac{\partial}{\partial x}, & x<0,\end{cases}$$

where c is the constant premium income rate. For the sake of simplicity we restrict our attention to exponentially distributed claim sizes, i.e. $P(x)=1-\mathrm{e}^{-rx}$ for $x>0$. Denote by $M(a;c;x)$ the confluent hypergeometric function, and $U(a;c;x)$ the confluent hypergeometric function of the second kind; see e.g. Magnus *et al*. (1966). We will also use the following notation to simplify the expressions:

$$\omega(\rho)=1-\frac{\lambda}{\rho},\qquad \delta(\rho)=1-\frac{\lambda}{\rho}-\frac{\alpha}{\rho},\qquad \mu_x(\rho)=rx+\frac{cr}{\rho},$$

$$M^*(\rho,x)=M(\omega(\rho);\delta(\rho);\mu_x(\rho)),\qquad M_1^*(\rho,x)=M(\omega(\rho);1+\delta(\rho);\mu_x(\rho)),$$

$$U^*(\rho,x)=U(\omega(\rho);\delta(\rho);\mu_x(\rho)),\qquad U_1^*(\rho,x)=U(\omega(\rho);1+\delta(\rho);\mu_x(\rho)).$$

Theorem 3.1. *Let $\alpha > 0$ and $-c/\rho_2 \le b < 0 < a < \infty$. Define*

$$V(x) = \mathbf{E}_x e^{-\alpha\tau_{a,b}},\ b < x < a.$$

(1) If $\lambda = \rho_1 = \rho_2 \equiv \rho$, then

$$V(x) = \begin{cases} A_1 e^{-rx} \displaystyle\int_{\mu_0(\rho)}^{\mu_x(\rho)} e^y y^{\alpha/\rho} dy + A_2 e^{-rx}, & 0 < x < a, \\ A_3 e^{-rx} \displaystyle\int_{\mu_x(\rho)}^{\mu_0(\rho)} e^y y^{\alpha/\rho} dy + A_4 e^{-rx}, & b < x \le 0, \end{cases}$$

where

$$A_1 = -A_3 = \frac{(\rho + \alpha + cr + br)e^{ra} - \rho r e^{rb}}{(\rho + \alpha + cr + br)\int_{\mu_0(\rho)}^{\mu_a(\rho)} e^y y^{\alpha/\rho} dy},$$

$$A_2 = A_4 = \frac{\rho r e^{br}}{\rho + \alpha + cr + br}.$$

(2) If $\lambda = \rho_1 \ne \rho_2$, and $\delta(\rho_2) \ne -n$, for $n = 0, 1, 2, \ldots$, then

$$V(x) = \begin{cases} e^{-rx}\left(A_5 \displaystyle\int_{\mu_0(\rho_1)}^{\mu_x(\rho_1)} e^y y^{\alpha/\rho_1} dy + A_6\right), & 0 < x < a, \quad (3.1) \\ e^{-rx}(A_7 M^*(\rho_2, x) + A_8 U^*(\rho_2, x)), & b < x \le 0, \quad (3.2) \end{cases}$$

where

$$A_5 = \frac{(\rho_1 + \alpha + cr + br)e^{ra} - \rho_1 r e^{rb}}{(\rho_1 + \alpha + cr + br)\int_{\mu_0(\rho_1)}^{\mu_a(\rho_1)} e^y y^{\alpha/\rho_1} dy},$$

$$A_6 = \frac{\rho_1 r e^{br}}{\rho_1 + \alpha + cr + br},$$

$$A_7 = \frac{r\omega(\rho_2)U(\omega(\rho_2)+1; \delta(\rho_2)+1; \mu_0(\rho_2))A_6 + r e^{\mu_0(\rho_1)}\mu_0(\rho_1)^{\alpha/\rho_1} U^*(\rho_2, 0)A_5}{rM^*(\rho_2, 0)U_1^*(\rho_2, 0) + \frac{\alpha r}{\rho_2\delta(\rho_2)} M_1^*(\rho_2, 0)U^*(\rho_2, 0)},$$

$$A_8 = \frac{A_6 - A_7 M^*(\rho_2, 0)}{U^*(\rho_2, 0)}.$$

(3) If $\lambda = \rho_2 \ne \rho_1$, and $\delta(\rho_1) \ne -n$, for $n = 0, 1, 2, \ldots$, then

$$V(x) = \begin{cases} e^{-rx}\left(A_9 M^*(\rho_1, x) + A_{10} U^*(\rho_1, x)\right), & 0 < x < a, \\ e^{-rx}\left(A_{11} \displaystyle\int_{\mu_x(\rho_2)}^{\mu_0(\rho_2)} e^y y^{\alpha/\rho_2} dy + A_{12}\right), & b < x \le 0, \end{cases}$$

where

$$A_{12} = \frac{\rho_2 r e^{br}}{\rho_2 + \alpha + cr + br},$$
$$A_9 = \frac{e^{ra} U^*(\rho_1, 0) - A_{12} U^*(\rho_1, a)}{M^*(\rho_1, a) U^*(\rho_1, 0) - M^*(\rho_1, 0) U^*(\rho_1, a)},$$
$$A_{10} = \frac{e^{ra} - A_9 M^*(\rho_1, a)}{U^*(\rho_1, a)},$$
$$A_{11} = \frac{r A_{10} U_1^*(\rho_1, 0) - A_9 \frac{r\alpha}{\rho_1 \delta(\rho_1)} M_1^*(\rho_1, 0) - r A_{12}}{r e^{\mu_0(\rho_2)} \mu_0(\rho_2)^{\alpha/\rho_2}}.$$

(4) If $\lambda \neq \rho_1 \neq \rho_2$, and both $\delta(\rho_1), \delta(\rho_2) \neq -n$, for $n = 0, 1, 2, \ldots$, then

$$V(x) = \begin{cases} e^{-rx}(A_{13} M^*(\rho_1, x) + A_{14} U^*(\rho_1, x)), & 0 < x < a, \\ e^{-rx}(A_{15} M^*(\rho_2, x) + A_{16} U^*(\rho_2, x)), & b < x \leq 0, \end{cases}$$

where A_i's are constants that are the solutions of the following system of linear equations

$$\begin{cases} A_{13} \dfrac{\alpha r}{\rho_1 \delta(\rho_1)} M_1^*(\rho_1, 0) - A_{14} r U_1^*(\rho_1, 0) = A_{15} \dfrac{\alpha r}{\rho_2 \delta(\rho_2)} M_1^*(\rho_2, 0) - A_{16} r U_1^*(\rho_2, 0), \\ A_{13} M^*(\rho_1, 0) + A_{14} U^*(\rho_1, 0) = A_{15} M^*(\rho_2, 0) + A_{16} U^*(\rho_2, 0), \\ A_{13} M^*(\rho_1, a) + A_{14} U^*(\rho_1, a) = e^{ra}, \\ A_{13} G_1 + A_{14} G_2 = A_{15} G_3 + A_{16} r^2 U(\omega(\rho_2), 2 + \delta(\rho_2), \mu_0(\rho_2)), \end{cases}$$

where

$$G_1 = \frac{\alpha r^2 (\omega(\rho_1) - \delta(\rho_1) - 1)}{\rho_1 \delta(\rho_1)(1 + \delta(\rho_1))} M(\omega(\rho_1), 2 + \delta(\rho_1), \frac{cr}{\rho_1}) - \frac{\alpha r (\rho_2 - \rho_1)}{c \rho_1 \delta(\rho_1)} M_1^*(\rho_1, 0),$$
$$G_2 = r^2 U(\omega(\rho_1); 2 + \delta(\rho_1); \mu_0(\rho_1)) + \frac{r(\rho_2 - \rho_1)}{c} U_1^*(\rho_1, 0),$$
$$G_3 = \frac{\alpha r^2 (\omega(\rho_1) - \delta(\rho_1) - 1)}{(\rho_1 - \lambda - \alpha)(1 + \delta(\rho_1))} M(\omega(\rho_2); 2 + \delta(\rho_2); \mu_0(\rho_2)).$$

Proof. By Lemma 1.1, $V(\cdot)$ is absolute continuous in (b, a) and satisfies (1.1) with the boundary condition $V(a) = 1$. Multiplying (1.1) by e^{-rx} and differentiating with respect to x yield

$$(\rho x + c) V''(x) + (\rho r x + rc + \rho - \lambda - \alpha) V'(x) - \alpha r V(x) = 0, \tag{3.3}$$

where $\rho = \rho_1$ if $0 < x < a$ and $\rho = \rho_2$ if $b < x \leq 0$. We only prove case (2) here; the proofs of cases (1), (3) and (4) are similar.

If $\lambda = \rho_1$, then the bounded solution of (3.3) for $0 < x < a$ is of the form of (3.1), where A_5 and A_6 are constants.

If $\lambda \neq \rho_2$, substituting $W(y) = \mathrm{e}^{y - cr/\rho_2} V(\frac{y}{r} - \frac{c}{\rho_2})$ into (3.3) yields

$$yW''(y) + \left(1 - \frac{\alpha}{\rho_2} - \frac{\lambda}{\rho_2} - y\right) W'(y) + \left(\frac{\lambda}{\rho_2} - 1\right) W(y) = 0, \quad \frac{cr}{\rho_2} + br < y < \frac{cr}{\rho_2},$$

which is Kummer's differential equation, see e.g. Magnus *et al.* (1966). If $\delta(\rho_2) \neq -n$, for $n = 0, 1, 2, \ldots$, the complete solution is given by

$$W(y) = A_7 M(\omega(\rho_2); \delta(\rho_2); y) + A_8 U(\omega(\rho_2); \delta(\rho_2); y), \quad \frac{cr}{\rho_2} + br < y < \frac{cr}{\rho_2},$$

where A_7 and A_8 are constants. Thus, equation (3.2) follows. The values of A_5, A_7 and A_8 can be determined by the boundary condition $V(a) = 1$ and the continuity of $V(\cdot)$ and $V'(\cdot)$ at zero. Finally, substituting (3.1) into (1.1) and equating the coefficients of e^{-rx} yield the expression for A_6.

Remark 3.1. Letting $a \to \infty$ and taking $b = -c/\rho_2$, we can obtain the Laplace transform of the distribution of the time of absolute ruin, and from which the probability of absolute ruin in the infinite horizon case can be derived.

References

Ablowitz, M. J., Fokas, A. S., 1997. Complex Variables: Introduction and Applications. Cambridge University Press, Cambridge.

Asmussen, S., 2000. Ruin Probabilities. World Scientific, Singapore.

Dassios, A., Embrechts, P., 1989. Martingales and insurance risk. Commun. Statist.–Stochastic Models 5, 181-217.

Davis, M. H. A., 1993. Markov Models and Optimization. Chapman and Hall, London.

Embrechts, P., Schmidli, H., 1994. Ruin estimation for a general insurance risk model. Adv. Appl. Probab. 26, 404-422.

Magnus, W., Oberhettinger, F., Soni, R. P., 1966. Formulas and Theorems for the Special Functions of Mathematical Physics. Springer-Verlag, New York.

Rolski, T., Schmidli, H., Schmidt, V., Teugels, J., 1999. Stochastic Processes for Insurance and Finance. John Wiley & Sons, New York.