

MASTER'S THESIS

On lower bounds of mixture L_2 -discrepancy, construction of uniform design and gamma representative points with applications in estimation and simulation

Ke, Xiao

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Abstract

Two topics related to the experimental design are considered in this thesis. On the one hand, The uniform experimental design (UD), a major kind of space-filling design, is widely used in applications. The majority of UD tables (UDs) with good uniformity are generated under the centralized L_2 -discrepancy (CD) and the wrap-around L_2 -discrepancy (WD). Recently, the mixture L_2 -discrepancy (MD) is proposed and shown to be more reasonable than CD and WD in terms of uniformity. In first part of the thesis we review lower bounds for MD of two-level designs from a different point of view and provide a new lower bound. Following the same idea we obtain a lower bound for MD of three-level designs. Moreover, we construct UD tables under the measurement of MD by the threshold accepting (TA) algorithm, and finally we attach two new UD tables with good properties derived from TA under the measurement of MD. On the other hand, the problem of selecting a specific number of representative points (RPs) to maintain as much information as a given distribution has raised attention. Previously, a method has been given to select type-II representative points (RP-II) from normal distribution. These point sets have good properties and minimize the information loss. Whereafter, following similar idea, Fu, 1985 have discussed RP-II for gamma distribution. In second part of the thesis, we improve the discussion of selecting Gamma RP-II and provide more RP-II tables with a number of parameters. Further in statistical simulation, we also evaluate the estimation performance of point sets resampled from Gamma RP-II by making comparison in different situations.

Keywords: Hamming distance, Mixture discrepancy, Threshold accepting algorithm, Uniform design, Representative points, Loss function, Gamma distribution, Resampling, Moment estimation, Maximum likelihood estimation

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Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	vi
List of Figures	viii
Chapter 1 Introduction	1
1.1 Overview	1
1.2 Preliminary and notations	2
1.2.1 Uniform experimental design	2
1.2.2 Representative points	7
1.3 Summary of the dissertation	8
Chapter 2 Lower bounds for MD of Two- and three-level designs	10
2.1 Lower bound for MD of two-level designs	10
2.2 Lower bound for MD of three-level designs	12
Chapter 3 Construction of UD's using TA algorithm	17
3.1 Effect from the selection of initial design	19
3.2 Iteration formulae	21
3.3 Uniform designs generated by TA under MD	22

Chapter 4	Selecting a specified number of RP-II from Gamma distribution	25
4.1	Mathematical model	25
4.2	Properties for the function set	27
4.3	Algorithm to obtain Gamma RP-II	32
Chapter 5	Application of Gamma RP-II in estimation and simulation	34
5.1	Estimating Gamma parameters from RP-II	34
5.2	Application of Gamma RP-II in statistical simulation	39
5.2.1	Estimation in simulation based on RP-II, RP-I and Gamma distribution	42
Chapter 6	Conclusion and further study	47
	Bibliography	49
	Curriculum Vitae	60