

MASTER'S THESIS

Is the precision of computed solutions more closely related with componentwise condition number than normwise condition number?

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Abstract

We have a conjecture that “the precision of computed solutions for systems of linear equations is more closely related with componentwise condition number $c(A)$ than normwise condition number $\kappa(A)$ ”. We conducted simulation experiments to verify this conjecture. A statistical tool, Hotelling-Williams T-Test is employed to check if difference between correlations is significant. Simulation results suggest that our conjecture is true for most of the well-known methods and matrix sizes.

Keywords: condition numbers, simulation, correlation coefficients, Hotelling-Williams T-Test

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Table of Contents

| | |
|---|-------------|
| Declaration | i |
| Abstract | ii |
| Acknowledgements | iii |
| Table of Contents | iv |
| List of Tables | vii |
| List of Figures | viii |
| Chapter 1 Introduction | 1 |
| 1.1 Conjecture, goal, suggestion and value of this paper | 3 |
| 1.2 Organization of this paper | 3 |
| Chapter 2 Simulation methods | 5 |
| 2.1 Probability model | 5 |
| 2.2 Computing condition numbers | 7 |
| 2.3 Computing Losses of Precision | 7 |
| 2.4 Methods for solving systems of linear equations | 8 |
| 2.4.1 (MLD) MATLAB command - mldivide and (GEP) Gaussian Elimination Method with partial pivoting | 8 |

| | | |
|--|--|-----------|
| 2.4.2 | (GEM) Gaussian Elimination Method without partial pivoting | 10 |
| 2.4.3 | (QRD) QR Decomposition | 10 |
| 2.4.4 | (CGM) Conjugate Gradient Method | 11 |
| 2.5 | Computing the precision of computed solutions | 12 |
| 2.6 | Population correlation coefficients | 13 |
| 2.7 | Sample correlation coefficients | 13 |
| 2.8 | Restate our conjectures | 14 |
| 2.9 | Accuracy of computed correlations and sample size | 14 |
| Chapter 3 Simulation results | | 16 |
| 3.1 | Average precisions of solutions computed by different methods . . | 18 |
| 3.2 | MATLAB command - mldivide (MLD) | 18 |
| 3.3 | Gaussian Elimination Method with partial pivoting (GEP) | 19 |
| 3.4 | Gaussian Elimination Method without partial pivoting (GEM) . . | 22 |
| 3.5 | QR Decomposition (QRD) | 22 |
| 3.6 | Conjugate Gradient Method (CGM) | 24 |
| 3.7 | Summarizing our simulation result suggestions | 25 |
| Chapter 4 Explanation for the small difference between the two correlation coefficients | | 26 |
| 4.1 | Statistical Method For Correlation Coefficients Comparison | 28 |
| 4.2 | Hypothesis Testing of MLD | 29 |
| 4.3 | Hypothesis Testing of GEP | 30 |
| 4.4 | Hypothesis Testing of GEM | 30 |
| 4.5 | Hypothesis Testing of QRD | 31 |
| 4.6 | Hypothesis Testing of CGM | 32 |
| Chapter 5 Conclusion | | 33 |

| | |
|------------------|----|
| Bibliography | 34 |
| Curriculum Vitae | 35 |