

MASTER'S THESIS

Is the precision of computed solutions more closely related with componentwise condition number than normwise condition number?

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Date of Award:
2015

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Abstract

We have a conjecture that “the precision of computed solutions for systems of linear equations is more closely related with componentwise condition number $c(A)$ than normwise condition number $\kappa(A)$ ”. We conducted simulation experiments to verify this conjecture. A statistical tool, Hotelling-Williams T-Test is employed to check if difference between correlations is significant. Simulation results suggest that our conjecture is true for most of the well-known methods and matrix sizes.

Keywords: condition numbers, simulation, correlation coefficients, Hotelling-Williams T-Test

Acknowledgements

I would like to express my gratitude to all those who helped me during the writing of this thesis. I gratefully acknowledge the help of my supervisor, Dr. Dennis Cheung, who has given me valuable suggestions in these two years of studies. In the preparation of the thesis, he has spent much time to read and check each draft. Without his patient instruction and expert guidance, the completion of this thesis would not be possible.

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