DOCTORAL THESIS

Finite element methods for some elliptic problems with singularity and problems on unbounded domains

Jin, Jicheng

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Finite Element Methods for Some Elliptic Problems with Singularity and Problems on Unbounded Domains

JIN Jicheng

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Abstract

Finite element method is the most important numerical technique for partial differential equations. This thesis is mainly concerned with the application of finite element method to the elliptic problems with interfaces, the Stokes equations with corner singularities and the one- and two-dimensional time-dependent Schrödinger equations on unbounded domains.

For problems with singular solutions, the standard finite element method is difficult to give satisfactory numerical results due to the presence of the singularities. In order to increase the accuracy of numerical solutions, several improved methods have been proposed, which includes "local mesh refinement method", "singular point separation method" and "singular finite element method".

One of the main contribution of the thesis is to develop the singular finite element method by eliminating the requirement for the analytical singular expansion form of the exact solution. We firstly obtain a discrete singular expansion near singular point by solving a simple eigenvalue problem, which is one dimension less than the original problem, and then use the approximation of the singularity together with the standard finite element basis functions to construct the singular finite element space. Finally, we find the singular finite element solution on a conventional mesh. Our method has been successfully applied to our concerned problems in the thesis. Numerical examples show that this method is very effective.

Another main contribution of the thesis is to apply finite element method successfully to the one- and two-dimensional time-dependent Schrödinger equations on unbounded domains. So far, there has been a lot of work on the numerical approximation of the above problem by finite difference method. But to our knowledge, the application of finite element method to this problem has been much less studied. Particularly, no theoretical analysis of finite element approximation has been discussed. In the thesis, we firstly reduce original problem into an initial-boundary value problem on a bounded domain by introducing a exact artificial boundary condition, and then fully discretize this reduced problem by applying Crank-Nicolson scheme in time and linear or quadratic finite element approximation in space. This scheme has proved to be unconditionally stable and convergent. Numerical examples have verified our theoretical results.

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