DOCTORAL THESIS

Moving mesh methods for singular problems in two dimensions

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Date of Award:
2004

Moving Mesh Methods for Singular Problems in Two Dimensions

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A thesis submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

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December 2004

Abstract

Adaptive mesh methods have important applications for a variety of physical and engineering problems. The physical phenomena for these problems develop dynamically singular or nearly singular solutions in fairly localized regions. With uniform meshes, the amount of computational time is too large to enable us to obtain useful numerical approximations, particular in multi-dimensions. Therefore, developing effective and robust moving mesh methods for these problems becomes necessary. In this thesis, we will study moving mesh methods for solving singular problems.

We develop efficient moving mesh algorithm for two-dimensional problems. Since the diffusion terms are involved, the underlying partial differential equations are transformed to the computational domain via a coordinate transformation. The resulting transformed partial differential equations are then solved in the computational domain equipped with a fixed uniform mesh.

Our moving mesh algorithm is an extension of Tang and Tang's [31] recent work for hyperbolic conservation laws and Zhang's [35] thesis for convection-dominated equations and nonlinear conservation laws. In this thesis, we also investigate the application of moving mesh techniques for some of the problems such as blow up problems and the Boussinesq convection problem. These are challenging problems both numerically and analytically. Numerical results demonstrate the advantage of our moving mesh method in resolving the small structures.

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