

## DOCTORAL THESIS

### Moving mesh finite volume method and its applications

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# Moving Mesh Finite Volume Method and its Applications

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for the degree of  
Doctor of Philosophy

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# Abstract

Moving mesh method has become an important numerical tool for computing singular or nearly singular problems arising from a variety of physical and engineering areas. This thesis is mainly concerned with the application of moving mesh method to nonlinear conservation laws, convection-diffusion equations, phase-field equations and magneto-hydrodynamics (MHD) model problems.

Our moving mesh algorithm is an extension of Tang's recent work which focuses on hyperbolic conservation laws. We have written the underlying PDEs in the computational domain via a coordinate transformation, and then the transformed PDEs are solved in computational domain equipped with fixed uniform mesh. Effectiveness and robustness of the proposed algorithm are demonstrated by numerical experiments.

It is well known that the time steps associated with the moving mesh methods are proportional to the smallest mesh size in space in order to guarantee stability. Since the moving mesh method is useful for problems whose solutions are singular in fairly localized regions, it reduces the allowable time step only in small part of the solution domain. It is then natural to use locally varying time steps to enhance the efficiency of the moving mesh methods. We design an efficient local time stepping scheme for the nonlinear hyperbolic conservation laws and the convection-dominated problems.

Finally we apply moving mesh method to the phase-field equations and magneto-hydrodynamics (MHD) model. Numerical results demonstrate the advantages of our moving mesh method in solving these problems.

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