DOCTORAL THESIS

Combinatorial properties of uniform designs and their applications in the constructions of low-discrepancy designs

Tang, Yu

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Combinatorial Properties of Uniform Designs and Their Applications in the Constructions of Low-discrepancy Designs

TANG Yu

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Principal Supervisors:
Prof. FANG Kai-Tai & Prof. YIN Jianxing

Hong Kong Baptist University

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Abstract

Uniform design has been widely applied in many fields, such as manufacturing, system engineering, pharmaceutics and natural sciences since it appears in 1980's. However, the theoretical parts concerned about uniform design including many essential properties as well as connections with other designs have not been extensively discussed until recently. As it is pointed out in many literatures, the most practical success for uniform design is due to its ability to investigate lots of high level factors simultaneously with fairly economical experimental runs. In recent years, the associated questions of construction and theoretical properties of uniform designs are interesting again due to the complex and rich structures rendered by the many choices of designs and the various purposes of the experiments. Such systematic discussion about uniform design will definitely make its more applications to economic and social problems possible.

Uniform design suggests choose a set of points over a certain domain such that these points are uniformly scattered. The measure of uniformity plays a key role in constructing uniform design. As modifications to the L_p-discrepancy, many discrepancies such as the discrete discrepancy, the wrap-around L_2-discrepancy and the centered L_2-discrepancy have been proposed. These criteria can not only be regarded as single values indicating the computational results for the corresponding designs. In fact, all of these discrepancies have their own geometrical explanations. These geometrical interpretations require uniform designs which achieve the minimal values of the corresponding discrepancy should have certain combinatorial properties. The first part of this thesis will discuss these combinatorial structures based on the discrete discrepancy, the wrap-around L_2-discrepancy and the centered L_2-discrepancy, respectively.

To search a uniform design has long been regarded as an NP hard problem with respect to the number of runs, factors and levels. Due to the complexity of the computational capacity, most of the known existing uniform designs have number of runs less than 50, as listed in the web site "http://www.math.hkbu.edu.hk/UniformDesign/". The second part of this thesis will provide two types of construction methods for uniform designs and low-discrepancy designs. These methods fully utilize the combinatorial properties in the first part of the thesis, thus make both the directive combinatorial constructions and the computational optimization approaches efficient. A lot of infinite classes of uniform designs are produced by using the known or new defined combinatorial configurations. Many moderate size designs with satisfactory low discrepancy are found by implementing efficient stochastic optimization algorithms.

The third part of this thesis provides a general concept of the discrepancy, which is defined by using reproducing kernel or covariance kernel. Under certain restrictions, lower bounds of some special types of discrepancies as well as combinatorial characteristic of designs achieving the lower bounds are discussed. As an application, the equivalence conjecture proposed in Fang, Lin, Winker and Zhang (2000) can then be fully solved.

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