DOCTORAL THESIS

Circular chromatic numbers and distance two labelling numbers of graphs

Lin, Wensong

Date of Award:
2004

Circular Chromatic Numbers and Distance Two Labelling Numbers of Graphs

LIN Wensong

A thesis submitted in partial fulfillment of the requirments
for the degree of
Doctor of Philosophy

Principal Supervisor: Dr. Peter C. B. LAM

Hong Kong Baptist University

October 2004

Abstract

Motivated by various practical and theoretical problems, a lot of variations and generalizations of the classic coloring problem have been developed. The applications of graph coloring theory to many other fields also have been studied extensively. The circular chromatic number of a graph is a refinement of the chromatic number of a graph and so reveals more information about the structure of the graph. The definition of $L(2,1)$-labelling number comes from a variation of channel assignment problem. When we assign colors to vertices of a graph, we not only consider the constraints of adjacent vertices but also the constraints of vertices at distance two. These two concepts were proposed at the end of 1980s. Both have significant applications to practical problems. Many aspects of these two graph parameters have been investigated. And there are so many problems concerning these two concepts remaining to be explored.

This thesis consists of two parts. In Part I, we focus on investigating the behavior of the circular chromatic numbers of several classes of special graphs. We obtain many star extremal circulant graphs and give a necessary and sufficient condition for a vertex-transitive graph to be star extremal. The circular chromatic numbers of distance graphs with distance sets missing an interval are also studied extensively in this thesis. And we determine the circular chromatic numbers of the generalized Mycielski graphs of cycles. In Part II, we deal with the $L(2,1)$-labelling numbers of graphs, particularly on products of graphs. The $L(2,1)$-labelling numbers of direct product of a complete graph and a path or two complete graphs are determined. Upper and lower bounds of the $L(2,1)$-labelling numbers of direct product of a complete graph and a cycle are also given. We design a polynomial time algorithm to determine whether the $L(2,1)$-labelling numbers of the direct products of an arbitrary graph and K_2 with a loop is less than its vertex number or not. Finally, we completely determine the $L(j,k)$-labelling number and the circular-$L(j,k)$-labelling number of direct product of complete graphs.

Keywords: Chromatic number, Circular chromatic number, Fractional chromatic number, Circulant graph, Distance graph, Mycielski graph, $\lambda_{2,1}$-number, $\lambda_{j,k}$-number, $\sigma_{j,k}$-number, Cartesian product of graphs, Direct product of graphs etc.

Table of Contents

Declaration i

Abstract ii

Acknowledgements iii

Table of Contents iv

List of Figures vi

List of Tables vii

Chapter 1 Introduction 1

I Circular chromatic number 4

Chapter 2 Star extremality for graphs 9

2.1 Star extremal graphs with $\chi_c(G) = \frac{|V(G)|}{\alpha(G)}$ 11

2.2 Some star extremal circulant graphs 16

Chapter 3 Colorings of distance graphs 24

3.1 Coloring problems of distance graphs 24

3.2 $G(Z, D_{m,[k,sk+\beta]})$ with $s = 1$ and $1 \leq \beta \leq k-1$ 27

3.3 $G(Z, D_{m,[k,sk+\beta]})$ with $s \geq 2$ 36

3.3.1 About $G(Z, D_{m,[k,sk+\beta]})$ for $m < (s+1)k+\beta$ 36

3.3.2 About $G(Z, D_{m,[k,sk+\beta]})$ for $m \geq (s+2)k+\beta-1$ 37

3.3.3 For $(s+1)k+\beta \leq m \leq (s+2)k+\beta-2$ 44

3.4 χ_c and χ_f of $G(Z, D_{[l,m],k,s})$ 49

Chapter 4 Circular chromatic numbers of the generalized Mycielski's graphs **54**

4.1 Introduction 54

4.2 Circular chromatic numbers of the generalized Mycielskians of cycles 55

II Distance two labelling number **63**

Chapter 5 $L(2,1)$-labellings of direct products of graphs **67**

5.1 Preliminaries 67

5.2 $\lambda(K_m \times K_n)$ 69

5.3 $\lambda(K_m \times P_n)$ 70

5.4 $\lambda(K_m \times C_n)$ 78

Chapter 6 Claw matching and $L(2,1)$-labelling **82**

6.1 t-claw-matching 83

6.2 λ-numbers of $\mu(G)$ and $G \times \hat{K}_2$ 86

6.3 λ-numbers of $\mu(K_n)$ and $K_n \times \hat{K}_2$ 88

Chapter 7 $L(j,k)$-labellings and circular $L(j,k)$-labellings of products of graphs **90**

7.1 $\lambda_{j,k}(K_m \times K_n)$ 91

7.2 $\sigma_{j,k}(K_m \times K_n)$ and $\sigma_{j,k}(K_m \Box K_n)$ 97

Bibliography **104**

Curriculum Vitae **114**