DOCTORAL THESIS

Distance-two constrained labellings of graphs and related problems

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Abstract

This thesis deals with distance-two constrained labelling of graphs which arises in the context of frequency assignment problem in mobile and wireless networks. The frequency assignment problem was first formulated as a graph coloring problem by Hale, who introduced the notion of the T-coloring of a graph, and then has been of particular interest for the graph coloring. In 1988, Roberts proposed a variation of the channel assignment problem in which "close" transmitters must receive different channels and "very close" transmitters must receive channels at least two apart. Motivated by this variation, Griggs and Yeh first proposed and studied the $L(2,1)$-labelling of a simple graph. Because of practical and theoretical applications, the interest for distance-two constrained labelling of graphs is increasing and since then, also many aspects of related problems remain to be explored.

In this thesis, we first give the values of the $L(2,1)$-labelling numbers of special graphs such as compositions of graphs, power paths, power cycles and the coronas of cycles, paths and complete graphs. Then we characterize unit interval graphs with given $L(2,1)$-labelling number. Some necessary and sufficient conditions for unit interval graph to have some given λ-number are obtained.

Concerning the long standing conjecture by Griggs and Yeh, which states that the λ-number of a graph cannot exceed the square of its maximum degree, we investigate the distance-two labelling of distance graphs which has not been previously investigated. The upper and lower bounds of λ-number for distance graphs are established. One significant result obtained is the truth of the Δ^2-conjecture for distance graphs. We discuss the periodic labellings of distance graphs analogous to the periodic colorings and prove that there exists an algorithm to determine the λ-number for any distance graph with distance set D of finite positive integers. For some special distance sets, better upper bounds are obtained. Especially, the exact values of λ-numbers for some 2-element distance sets are determined by introducing the generalized structure of Cartesian product of path and cycle.

As a related problem, the full $L(2,1)$-colorability of graphs is studied. We show that there always exists a connected graph G with an arbitrary pair of λ-number and hole index. In addition, we discuss the near λ-optimality of some graphs such as some subclasses of bipartite and outerplanar graphs. Two questions proposed by Fishburn and Roberts are settled.

For another related problem, the labelling extension problem in which some vertices of a graph have been pre-labelled, is also treated. For general graphs, we obtain an upper bound for the minimum span of the distance two labelling extending any pre-labelling. We improve an upper bound for t-degenerate graphs and prove that a conjecture holds for 2-degenerate graphs, which was posed by Bodlaender et al. in 2002.

Finally, we introduce a convex labelling of simple graphs as a special distance-two labelling, which is a natural generalization of the average labelling proposed by Harminc and Soták(2002). Then we characterize the graphs with any convex labelling and all the admissible convex labellings for such graphs. Our results include the results of Harminc and Soták for average labellings.

Keywords: Chordal graph, Distance graph, Distance-two constrained labelling, λ-number, Full $L(2,1)$-colorability, No-hole labelling, Pre-labelling, Convex labelling etc.

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