

DOCTORAL THESIS

Distance-two constrained labellings of graphs and related problems

Gu, Guohua

Date of Award:
2005

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

Distance-Two Constrained Labellings of Graphs and Related Problems

GU Guohua

A thesis submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

Principal Supervisor: Dr. Peter C.B. LAM

Hong Kong Baptist University

March 2005

Abstract

This thesis deals with distance-two constrained labelling of graphs which arises in the context of frequency assignment problem in mobile and wireless networks. The frequency assignment problem was first formulated as a graph coloring problem by Hale, who introduced the notion of the T-coloring of a graph, and then has been of particular interest for the graph coloring. In 1988, Roberts proposed a variation of the channel assignment problem in which “close” transmitters must receive different channels and “very close” transmitters must receive channels at least two apart. Motivated by this variation, Griggs and Yeh first proposed and studied the $L(2, 1)$ -labelling of a simple graph. Because of practical and theoretical applications, the interest for distance-two constrained labelling of graphs is increasing and since then, also many aspects of related problems remain to be explored.

In this thesis, we first give the values of the $L(2, 1)$ -labelling numbers of special graphs such as compositions of graphs, power paths, power cycles and the coronas of cycles, paths and complete graphs. Then we characterize unit interval graphs with given $L(2, 1)$ -labelling number. Some necessary and sufficient conditions for unit interval graph to have some given λ -number are obtained.

Concerning the long standing conjecture by Griggs and Yeh, which states that the λ -number of a graph cannot exceed the square of its maximum degree, we investigate the distance-two labelling of distance graphs which has not been previously investigated. The upper and lower bounds of λ -number for distance graphs are established. One significant result obtained is the truth of the Δ^2 -conjecture for distance graphs. We discuss the periodic labellings of distance graphs analogous to the periodic colorings and prove that there exists an algorithm to determine the λ -number for any distance graph with distance set D of finite positive integers. For some special distance sets, better upper bounds are obtained. Especially, the exact values of λ -numbers for some 2-element distance sets are determined by introducing the generalized structure of Cartesian product of path and cycle.

As a related problem, the full $L(2, 1)$ -colorability of graphs is studied. We show that there always exists a connected graph G with an arbitrary pair of λ -number and hole index. In addition, we discuss the near λ -optimality of some graphs such as some subclasses of bipartite and outerplanar graphs. Two questions proposed by Fishburn and Roberts are settled.

For another related problem, the labelling extension problem in which some vertices of a graph have been pre-labelled, is also treated. For general graphs, we obtain an upper bound for the minimum span of the distance two labelling extending any pre-labelling. We improve an upper bound for t -degenerate graphs and prove that a conjecture holds for 2-degenerate graphs, which was posed by Bodlaender et al. in 2002.

Finally, we introduce a convex labelling of simple graphs as a special distance-two labelling, which is a natural generalization of the average labelling proposed by Harminc and Soták(2002). Then we characterize the graphs with any convex labelling and all the admissible convex labellings for such graphs. Our results include the results of Harminc and Soták for average labellings.

Keywords: Chordal graph, Distance graph, Distance-two constrained labelling, λ -number, Full $L(2, 1)$ -colorability, No-hole labelling, Pre-labelling, Convex labelling etc.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iv
Table of Contents	v
List of Figures	vii
Chapter 1 Introduction	1
1.1 Frequency Assignment Problem	1
1.2 Graph Labelling Problems	3
1.3 Outline of the Thesis	7
Chapter 2 Labelling Several Classes of Special Graphs	9
2.1 An Upper Bound for λ	9
2.2 Compositions of Graphs	12
2.3 Power Paths and Power Cycles	15
2.4 Coronas of Cycles, Paths and Complete Graphs	21
Chapter 3 Distance Two Labelling Characterization of Unit Interval Graphs	25
3.1 Some Notations	26
3.2 Characterization of Unit Interval Graphs with $\lambda = 2\chi - 2$	26
3.3 Substructures of Unit Interval Graphs with $\lambda = 2\chi$	40
3.4 Concluding Remarks	44

Chapter 4	Labelling Distance Graphs	46
4.1	Basic Definitions and Results	47
4.2	Periodic Labellings of Distance Graphs	51
4.3	Finite Distance Sets	53
4.4	Two Element Distance Sets	56
Chapter 5	Full $L(2, 1)$-Colorability of Graphs	64
5.1	Introduction	64
5.2	Notations and Preliminaries	66
5.3	Graphs with Arbitrary Hole Indices	69
5.4	Near $L(2, 1)$ -Optimality	73
5.5	Aspects of $C_4(5)$	78
Chapter 6	Prelabelling Extension of Graphs	82
6.1	Notations and Preliminaries	83
6.2	$L(1, 1)$ -Labelling Extension	84
6.3	$L(2, 1)$ -Labelling Extension	86
6.4	$L'(2, 1)$ -Labelling Extension	88
Chapter 7	Convex Labellings of Graphs	99
7.1	Introduction	99
7.2	Preliminary Lemmas	101
7.3	Graphs with Nontrivial Convex Labellings	103
7.4	Nontrivial Convex Labellings	114
Curriculum Vitae		130