

DOCTORAL THESIS

Fractional differential equations for modelling financial processes with jumps

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Date of Award:
2015

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ABSTRACT

The standard Black-Scholes model is under the assumption of geometric Brownian motion, and the log-returns for Black-Scholes model are independent and Gaussian. However, most of the recent literature on the statistical properties of the log-returns makes this hypothesis not always consistent. One of the ongoing research topics is to find a better financial pricing model instead of the Black-Scholes model.

In the present work, we concentrate on two typical 1-D option pricing models under the general exponential Lévy processes, namely the finite moment log-stable (FMLS) model and the Carr-Geman-Madan-Yor-eta (CGMYe) model, and we also propose a multivariate CGMYe model. Both the frameworks, and the numerical estimations and simulations are studied in this thesis.

In the future work, we shall continue to study the fractional partial differential equations (FPDEs) of the financial models, and seek for the efficient numerical algorithms of the American pricing problems.

Keywords: fractional partial differential equation; option pricing models; exponential Lévy process; approximate solution.

ACKNOWLEDGEMENTS

First and foremost, I would take this chance to express my deepest gratitude to Dr. LING Leevan, my principal supervisor, for his constant encouragement and guidance. Without his consistent illuminating instruction and great patience, this thesis could not have reached its present form. His professional quality, good habits in research field, and outstanding character in behaviors, will benefit me for life.

Meanwhile, I would also thank Prof. TANG Tao, my co-supervisor, and Prof. LIU Jianya of Shandong University, and Prof. Benny HON from the City University of Hong Kong, by this chance. Prof TANG, Prof. LIU and Prof. HON offered great supports and help at the most critical time in my life. Under their professional guidance, I took less detours on my research. I would thank Prof. WANG Hong and Dr. LI Yutian at the same time, for their invaluable advice and helpful comments in my study.

Moreover, I wish to thank Hong Kong Baptist University for providing quality courses and the whole-person education. I am also greatly indebted to the teachers, secretaries and technicians at the Department of Mathematics for their great help.

Finally, I wish to express my gratitude to my parents, for their love and confidence in me all through these years.

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