

## DOCTORAL THESIS

### Multiscale analysis by nonseparable wavelet and Hilbert-Huang transform

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# Multiscale Analysis by Nonseparable Wavelet and Hilbert-Huang Transform

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# Abstract

Multiscale analysis provides a promising framework for data analysis. Representing the real-world objects in different scales and then using the outputs for further processing is an essential part in computer vision, image processing, signal processing and biological vision. Among kinds of multiscale representation approaches, wavelet is an irreplaceable one for its utmost contributions in last decades. Another one is the Hilbert-Huang Transform (HHT) proposed by N.E.Huang in 1998, which is an adaptive data decomposition method especially for the non-stationary and nonlinear data. However, both the wavelet and HHT have some limitations in image processing. In this thesis, we focus on the studies related to these two multiscale analysis approaches.

As for the wavelet approach, a type of nonseparable bivariate wavelet filter banks is constructed in this thesis. Most current bivariate wavelet filter banks are separable, i.e., generated by the tensor products of one-dimensional wavelets. Therefore, directional limitations exist when they are utilized in image processing. Constructing nonseparable wavelet is a desirable task of wavelet developments. Nevertheless, it is difficult to find general solutions for the two construction conditions because they are both quadratic algebraic equations with multiple variables. In this thesis, we propose a class of solutions based on a type of desired centrally symmetric matrix. By adopting the centrally symmetric matrices, new non-tensor product bivariate wavelet filter banks with linear phase are constructed. These filter banks have a matrix factorization, and are capable of capturing more detailed feature information without limitation of orientations. Furthermore, we evaluate the facial feature extraction ability of the new nonseparable wavelet filter banks through applying them in face recognition.

As for the HHT approach, we focus its first part: Empirical Mode Decomposition (EMD), which is the major innovation of the HHT as well. There are main two aspect works in this thesis. First, a new fast bidimensional EMD (BEMD) method is proposed. The EMD approach decomposes a signal into different Intrinsic Mode Function (IMF) components with various frequency scales by a data-driven sifting

process. Due to the adaptiveness property, it is efficient in non-stationary and non-linear data analysis. One key problem in the sifting process is the mean component generated by the interpolation of extrema data points. Therefore, the interpolation method is crucial to determine the final performance and time consumption, especially in the bidimensional case. In this thesis, we propose an alternative BEMD method based on the average filters instead of surface interpolation. Thereinto, one crucial factor is the determination of the size of the average filter. In order to improve the stableness and performances of the proposed BEMD method, two adaptive window sizes are designed depending on the adjacent distance matrix, rather than the experimental experiences. Furthermore, we adopt our BEMD method in image analysis. The local features are detected from the monogenic signals which are obtained via the Riesz transform on the components generated by the new BEMD method.

The second work about the EMD approach is that we address the orthogonality problem. We prove that the EMD is nonorthogonal, then we propose two methods to generalize orthogonal EMD. Many literatures only check the orthogonality in practical sense rather than elaborate in theoretical sense. We first propose an alternative necessary condition for the orthogonality, and then prove the EMD is nonorthogonal via a counterexample. Furthermore, we develop two methods to create the orthogonal EMD (OEMD). One is the Empirical Mode Frequency Decomposition (EMFD), the other one is the orthogonalization of the original EMD. For the EMFD, we first design a series of ideal orthogonal filters to filter the data into different frequency components without overlapping. Back to the time domain, the corresponding signals are the desirable decomposition components named as Empirical Mode Frequency Decomposition Component (EMFDC), which are orthogonal to each other. One crucial problem of the proposed EMFD is the selection of proper frequency points for the ideal filters. An automatic selection criterion is established further to improve the performances. For the OEMD, we orthogonalize the IMFs obtained by the original EMD using the Gram-Schmidt orthogonalization method. Because the IMFs capture the information of the data from the finest scale to the coarsest scale, i.e., the highest frequency to the lowest frequency, two ways of orthogonalization can be developed. If we start from the first IMF to the last one, we call it High

Frequency Orthogonalization of the EMD (HF-OEMD), contrarily, we call it Low Frequency Orthogonalization of the EMD (LF-OEMD). This thesis also evaluates all the proposed EMD-based approaches in face recognition, especially in addressing illumination problem. Comparisons between the proposed approaches and the existing related algorithms are also performed and reported using three public available databases: CMU PIE, FERET and YaleB databases.

In short, the major contributions of this thesis are summarized as follows:

1. New nonseparable bivariate wavelet filter banks with linear phase are constructed. These filter banks are adopted to capture the illumination invariant facial features.
2. A new fast bidimensional Empirical Mode Decomposition (BEMD) method based on average filters is proposed. To improve the stableness and performance, two types of window size are designed depending on the adjacent distance matrix rather than practical experiences.
3. The proposed BEMD is applied in image analysis. The local features of the image are detected from the corresponding monogenic signals via realizing the Riesz transform on the generated bidimensional Intrinsic Mode Function (BIMF) components.
4. We prove that the EMD is nonorthogonal based on a defined necessary condition of orthogonality.
5. The Empirical Mode Frequency Decomposition (EMFD) method is proposed to get strict orthogonal IMF components. The kernel idea is utilizing a series of ideal orthogonal filters, where an automatic selection criteria of the frequency points is given.
6. Orthogonal EMD (OEMD) is proposed through an orthogonalization process of the original EMD. According to the order of starting IMF, two ways are available: High Frequency Orthogonalization of the EMD (HF-OEMD) and Low Frequency Orthogonalization of the EMD (LF-OEMD).

7. All the proposed EMD-based approaches including the new BEMD, EMFD, HF-OEMD and LF-OEMD are applied in face recognition. The performances and comparisons on the CMU PIE, FERET and YaleB databases are reported.

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