

DOCTORAL THESIS

The role of the counter rotating terms in spontaneous emission and the time evolution of lamb shift

Li, Zhenghong

Date of Award:
2012

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

**The Role of the Counter Rotating Terms in Spontaneous
Emission and the Time Evolution of Lamb Shift**

LI Zhenghong

A thesis submitted in partial fulfilment of the requirements

for the degree of

Doctor of Philosophy

Principal Supervisor: Prof. ZHU Shi Yao

Hong Kong Baptist University

December 2011

Abstract

The time-dependent effective decay rate and the time dependent Lamb shift in the spontaneous emission from a multi-level atom interacting with vacuum is investigated by a unitary transformation method, with which all rotating and counter-rotating terms in the Hamiltonian are included and the self-energy of the free electron, which is divergent, can be directly subtracted from the Hamiltonian. Consequently, the time evolution of the Lamb shift can be studied, which is impossible in traditional method of calculating. In addition, the ground state of the whole system (atom + vacuum modes) is easy to obtain. By starting with the ground state of the total Hamiltonian (not the ground state of the bare atom), the evolution of the effective decay rate of the atom (Zeno and anti-Zeno effects) is re-calculated.

The counter-rotating terms have important contribution to the dynamic evolution in the decay. When the atomic transition frequency is smaller than the central frequency of the coupling spectrum of the reservoir, the Zeno effect dominates, and if the atomic transition frequency is larger than the central frequency, the anti-Zeno effect will dominate.

The time evolution of the Lamb shift that accompanies the real photon emission is obtained. The Lamb shift can then be separated into two parts: one is the time-independent shift due to the virtual photon exchange, and the other is the time-dependent shift due to the real photon emission. The quantum Zeno and anti-Zeno effects for the Lamb shift is also found, which are dependent on the transition frequency and the coupling spectrum of the reservoir.

We also investigate the influence of the counter rotating terms and the Lamb shift on the quantum interference in spontaneous emission from a V-type three-level atom. The Lamb shift has great influence on the population decay for the two upper levels, as long as the transition dipoles are parallel and the energy separation of the two levels is much smaller than the transition frequencies. In this case, the photon emission and re-absorption (which leads to the Lamb shift) has an additional channel, the emitted from one transition and re-absorbed by another transition, besides by the same channel. This is the effect of the quantum interference due to the Lamb shift, which can be observed in the time scale of one over atomic decay rate, that is to say, experimentally observable.

The Lamb shift also has substantial influence on the spectrum emitted by the V-type atom. The Lamb shift makes the spectrum asymmetrical and moves the position of the dark line, which is obtained by comparing to the results without the Lamb shift (by neglecting all terms related to the Lamb shift). An experiment to test the effect of the quantum interference due to the Lamb shifts on the emission spectrum is suggested.

Table of Contents

Declaration.....	i
Abstract.....	ii
Acknowledgements.....	iii
Table of Contents	iv
List of Figures.....	vi
Chapter 1 Introduction	1
1.1 Spontaneous emission.....	1
1.2 Quantum Zeno Effect (QZE) and Quantum Anti-Zeno Effect (QAZE).....	2
1.3 The ground state of the total system due to the counter-rotating terms.....	4
1.4 Lamb Shift	6
1.5 Quantum interference	7
1.6 The structure of the thesis	9
Chapter 2 Basic Theory: The unitary transformation method	11
2.1 Introduction.....	11
2.2 The unitary transformation method	13
2.3 The coupling spectrum of the reservoir	21
2.4 The ground state of the total system	22
2.4.1 The time evolution of the state of the bare atom with no photon, $ g^H\rangle = g\rangle \{0_{\mathbf{k}}\}\rangle$	24
2.4.2 The energy of the $ g, \{0_{\mathbf{k}}\}\rangle$ state and the ground state $ G\rangle$	26
2.4.3 The energy of two excited states.....	27
2.5 Self-energy of the free electron.....	29
2.6 Summary.....	30
Chapter 3 Short time evolution: The QZE and QAZE due to the counter-rotating terms.....	31
3.1 Introduction.....	31

3.2 Dynamic equations	33
3.3 Effective decay rate.....	34
3.4 Summary	37
Chapter 4 Time evolution of Lamb shift.....	39
4.1 Introduction.....	39
4.2 The non-dynamic energy shift and the dynamic energy shift.....	41
4.3 The long time Lamb shift.....	44
4.4 Transient Lamb Shift	46
4.5 Summary.....	49
Chapter 5 Quantum interference due to the energy shifts.....	51
5.1 Introduction.....	51
5.2 The solution of the dynamic equations including the quantum interference.....	53
5.3 The influence of the energy shifts on the population evolution and the effective decay rates.....	58
5.4 The final energy of the two upper levels: Quantum interference pulls them together	62
5.5 The difference from Chapter 3: In what condition, the indirect interaction which results in the quantum interference should not be ignored?	65
5.6 Summary.....	70
Chapter 6 Spectrum of the field emitted by the atom.....	72
6.1 Introduction.....	72
6.2 The spontaneous emission spectrum of a two-level atom	73
6.3 The spontaneous emission spectrum of a V-type three-level atom.....	76
6.4 Summary	83
Conclusion	84
References.....	87
Curriculum Vitae.....	92