

## DOCTORAL THESIS

### The role of the counter rotating terms in spontaneous emission and the time evolution of lamb shift

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**The Role of the Counter Rotating Terms in Spontaneous  
Emission and the Time Evolution of Lamb Shift**

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## Abstract

The time-dependent effective decay rate and the time dependent Lamb shift in the spontaneous emission from a multi-level atom interacting with vacuum is investigated by a unitary transformation method, with which all rotating and counter-rotating terms in the Hamiltonian are included and the self-energy of the free electron, which is divergent, can be directly subtracted from the Hamiltonian. Consequently, the time evolution of the Lamb shift can be studied, which is impossible in traditional method of calculating. In addition, the ground state of the whole system (atom + vacuum modes) is easy to obtain. By starting with the ground state of the total Hamiltonian (not the ground state of the bare atom), the evolution of the effective decay rate of the atom (Zeno and anti-Zeno effects) is re-calculated.

The counter-rotating terms have important contribution to the dynamic evolution in the decay. When the atomic transition frequency is smaller than the central frequency of the coupling spectrum of the reservoir, the Zeno effect dominates, and if the atomic transition frequency is larger than the central frequency, the anti-Zeno effect will dominate.

The time evolution of the Lamb shift that accompanies the real photon emission is obtained. The Lamb shift can then be separated into two parts: one is the time-independent shift due to the virtual photon exchange, and the other is the time-dependent shift due to the real photon emission. The quantum Zeno and anti-Zeno effects for the Lamb shift is also found, which are dependent on the transition frequency and the coupling spectrum of the reservoir.

We also investigate the influence of the counter rotating terms and the Lamb shift on the quantum interference in spontaneous emission from a V-type three-level atom. The Lamb shift has great influence on the population decay for the two upper levels, as long as the transition dipoles are parallel and the energy separation of the two levels is much smaller than the transition frequencies. In this case, the photon emission and re-absorption (which leads to the Lamb shift) has an additional channel, the emitted from one transition and re-absorbed by another transition, besides by the same channel. This is the effect of the quantum interference due to the Lamb shift, which can be observed in the time scale of one over atomic decay rate, that is to say, experimentally observable.

The Lamb shift also has substantial influence on the spectrum emitted by the V-type atom. The Lamb shift makes the spectrum asymmetrical and moves the position of the dark line, which is obtained by comparing to the results without the Lamb shift (by neglecting all terms related to the Lamb shift). An experiment to test the effect of the quantum interference due to the Lamb shifts on the emission spectrum is suggested.

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