

## MASTER'S THESIS

# An ADMM approach to the numerical solution of state constrained optimal control problems for systems modeled by linear parabolic equations

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# Abstract

We address in this thesis the numerical solution of state constrained optimal control problems for systems modeled by linear parabolic equations. For the unconstrained or control-constrained optimal control problem, the first order optimality condition can be obtained in a general way and the associated Lagrange multiplier has low regularity, such as in the  $L^2(\Omega)$ . However, for state-constrained optimal control problems, additional assumptions are required in general to guarantee the existence and regularity of Lagrange multipliers. The resulting optimality system leads to difficulties for both the numerical solution and the theoretical analysis. The approach discussed here combines the alternating direction of multipliers (ADMM) with a conjugate gradient (CG) algorithm, both operating in well-chosen Hilbert spaces. The ADMM approach allows the decoupling of the state constraints and the parabolic equation, in which we need solve an unconstrained parabolic optimal control problem and a projection onto the admissible set in each iteration. It has been shown that the CG method applied to the unconstrained optimal control problem modeled by linear parabolic equation is very efficient in the literature. To tackle the issue about the associated Lagrange multiplier, we prove the convergence of our proposed algorithm without assuming the existence and regularity of Lagrange multipliers. Furthermore, a worst case  $O(1/k)$  convergence rate in the ergodic sense is established. For numerical purposes, we employ the finite difference method combined with finite element method to implement the time-space discretization. After full discretization, the numerical results we obtain validate the methodology discussed in this thesis.

**Keywords:** Optimal control problem, state constraint, ADMM algorithm, conjugate gradient method, convergence rate, finite difference method, finite element method

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