DOCTORAL THESIS

Numerical algorithms for data analysis with imaging and financial applications

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Abstract

In this thesis, we study modellings and numerical algorithms to data analysis with applications to image processing and financial forecast. The thesis is composed of two parts, namely the tensor regression and data assimilation methods for image restoration.

We start with investigating the tensor regression problem in Chapter 2. It is a generalization of a classical regression in order to adopt and analyze much more information by using multi-dimensional arrays. Since the regression problem is subject to multiple solutions, we propose a regularized tensor regression model to the problem. By imposing a low-rank property of the solution and considering the structure of the tensor product, we develop an algorithm which is suitable for scalable implementations. The regularization method is used to select useful solutions which depend on applications. The proposed model is solved by the alternating minimization method and we prove the convergence of the objective function values and iterates by the maximization-minimization (MM) technique. We study different factors which affects the performance of the algorithm, including sample sizes, solution ranks and the noise levels. Applications include image compressing and financial forecast.

In Chapter 3, we apply filtering methods in data assimilation to image restoration problems. Traditionally, data assimilation methods optimally combine a predictive state from a dynamical system with real partially observations. The motivation is to improve the model forecast by real observation. We construct an artificial dynamics to the non-blind deblurring problems. By making use of spatial information of a single image, a span of ensemble members is constructed. A two-stage use of ensemble transform Kalman filter (ETKF) is adopted to deblur corrupted images. The theoretical background of ETKF and the use of artificial dynamics by stage augmentation method are provided. Numerical experiments include image and video processing.

Concluding remarks and discussion on future extensions are included in Chapter 4.

Table of Contents

Declaration i

Abstract ii

Acknowledgements iii

Table of Contents iv

List of Figures vii

List of Tables x

Chapter 1 Overview of the thesis 1

1.1 Overview of tensor regression 1

1.1.1 Classical least squares problems 1

1.1.2 The tensor regression problem 2

1.1.3 Contribution 3

1.2 Overview of data assimilation to image restoration problems 4

1.2.1 Some commonly-used data assimilation methods 4

1.2.2 The image restoration problem 5

1.2.3 Contribution 5

1.3 Outline of the Thesis 6

Chapter 2 The tensor regression model 8

2.1 Matrix and tensor background 9

2.2 Related work 11

2.3 Proposed Algorithm 12

2.3.1 Choices for the regularization function 12
2.3.2 Alternating least squares 13
2.4 Optimization for the regularized model 14
2.5 Theoretical results 16
2.5.1 Surrogate functions 16
2.5.2 Convergence of function value 17
2.5.3 Quantification on difference of successive function values . . . 18
2.5.4 Convergence of iterates 21
2.5.5 Optimality of f 21
2.5.6 Convergence Rates 22
2.6 Numerical results 27
2.6.1 Synthetic experiments 27
2.6.1.1 Effect of solution ranks 27
2.6.1.2 Effect of sample sizes 28
2.6.2 Compress sensing by Gaussian samples 29
2.6.2.1 Effect of sample sizes 29
2.6.2.2 Effect of solution ranks 31
2.6.2.3 Effect of noise levels 32
2.6.3 Financial forecast 33
2.6.3.1 Training Part 33
2.6.3.2 Prediction Part 34
2.6.3.3 Stock results 36
2.6.3.4 Moving averages 45
2.7 Extended result: the tensor-matrix regression model 55
2.7.1 Solver for the tensor-matrix regression model 55
2.7.2 Preliminary results 56
2.8 Summary 63

Chapter 3 Data assimilation methods to image deblurring problems 64
3.1 The Difficulty of the Problem - the Inverted Noise 65
3.2 Data assimilation, notations and methods 66
3.2.1 Concepts and notations 66

3.2.2 The Kalman filter . . . 67
3.2.3 The ensemble Kalman filter . . . 70
3.2.4 The ensemble transform Kalman filter . . . 71
3.3 The ETKF to the image deblurring problem . . . 74
3.3.1 The augmented system . . . 74
3.3.2 Algorithm: ETKF to the image delburring problem . . . 75
3.3.3 Properties of the ETKF to the artificial dynamics . . . 76
3.3.3.1 Preliminary . . . 76
3.3.4 The Two-stage ETKF . . . 79
3.4 Numerical results . . . 81
3.4.1 Generation of the initial ensembles . . . 81
3.4.2 Image deblurring . . . 83
3.4.2.1 Experiment 1: Noise-free blurring model . . . 83
3.4.2.2 Experiment 2: Noise Blurring Model ($\sigma^2 = 0.5$) . . . 86
3.4.2.3 Experiment 3: Noise Blurring Model ($\sigma^2 = 1$) . . . 88
3.4.2.4 Experiment 4: Noise Blurring Model ($\sigma^2 = 1$); Two-stage ETKF . . . 90
3.4.3 Video deblurring . . . 92
3.4.3.1 Numerical result . . . 92
3.5 Summary . . . 93
3.6 Appendix . . . 94
3.6.1 EnKF without perturbation . . . 94
3.6.2 Optimization issue . . . 95

Chapter 4 Conclusion and future work 97
4.1 Conclusion . . . 97
4.2 Future work . . . 98
4.2.1 Discussion on the tensor regression problem . . . 98
4.2.2 Improvement on image restoration by the ensemble method . . . 98

Bibliography 100

Curriculum Vitae 105