

DOCTORAL THESIS

Numerical algorithms for data analysis with imaging and financial applications

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Abstract

In this thesis, we study modellings and numerical algorithms to data analysis with applications to image processing and financial forecast. The thesis is composed of two parts, namely the tensor regression and data assimilation methods for image restoration.

We start with investigating the tensor regression problem in Chapter 2. It is a generalization of a classical regression in order to adopt and analyze much more information by using multi-dimensional arrays. Since the regression problem is subject to multiple solutions, we propose a regularized tensor regression model to the problem. By imposing a low-rank property of the solution and considering the structure of the tensor product, we develop an algorithm which is suitable for scalable implementations. The regularization method is used to select useful solutions which depend on applications. The proposed model is solved by the alternating minimization method and we prove the convergence of the objective function values and iterates by the maximization-minimization (MM) technique. We study different factors which affects the performance of the algorithm, including sample sizes, solution ranks and the noise levels. Applications include image compressing and financial forecast.

In Chapter 3, we apply filtering methods in data assimilation to image restoration problems. Traditionally, data assimilation methods optimally combine a predictive state from a dynamical system with real partially observations. The motivation is to improve the model forecast by real observation. We construct an artificial dynamics to the non-blind deblurring problems. By making use of spatial information of a single image, a span of ensemble members is constructed. A two-stage use of ensemble transform Kalman filter (ETKF) is adopted to deblur corrupted images. The theoretical background of ETKF and the use of artificial dynamics by stage augmentation method are provided. Numerical experiments include image and video processing.

Concluding remarks and discussion on future extensions are included in Chapter 4.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Figures	vii
List of Tables	x
Chapter 1 Overview of the thesis	1
1.1 Overview of tensor regression	1
1.1.1 Classical least squares problems	1
1.1.2 The tensor regression problem	2
1.1.3 Contribution	3
1.2 Overview of data assimilation to image restoration problems	4
1.2.1 Some commonly-used data assimilation methods	4
1.2.2 The image restoration problem	5
1.2.3 Contribution	5
1.3 Outline of the Thesis	6
Chapter 2 The tensor regression model	8
2.1 Matrix and tensor background	9
2.2 Related work	11
2.3 Proposed Algorithm	12

2.3.1	Choices for the regularization function	12
2.3.2	Alternating least squares	13
2.4	Optimization for the regularized model	14
2.5	Theoretical results	16
2.5.1	Surrogate functions	16
2.5.2	Convergence of function value	17
2.5.3	Quantification on difference of successive function values	18
2.5.4	Convergence of iterates	21
2.5.5	Optimality of f	21
2.5.6	Convergence Rates	22
2.6	Numerical results	27
2.6.1	Synthetic experiments	27
2.6.1.1	Effect of solution ranks	27
2.6.1.2	Effect of sample sizes	28
2.6.2	Compress sensing by Gaussian samples	29
2.6.2.1	Effect of sample sizes	29
2.6.2.2	Effect of solution ranks	31
2.6.2.3	Effect of noise levels	32
2.6.3	Financial forecast	33
2.6.3.1	Training Part	33
2.6.3.2	Prediction Part	34
2.6.3.3	Stock results	36
2.6.3.4	Moving averages	45
2.7	Extended result: the tensor-matrix regression model	55
2.7.1	Solver for the tensor-matrix regression model	55
2.7.2	Preliminary results	56
2.8	Summary	63
Chapter 3 Data assimilation methods to image deblurring problems		64
3.1	The Difficulty of the Problem - the Inverted Noise	65
3.2	Data assimilation, notations and methods	66
3.2.1	Concepts and notations	66

3.2.2	The Kalman filter	67
3.2.3	The ensemble Kalman filter	70
3.2.4	The ensemble transform Kalman filter	71
3.3	The ETKF to the image deblurring problem	74
3.3.1	The augmented system	74
3.3.2	Algorithm: ETKF to the image delburring problem	75
3.3.3	Properties of the ETKF to the artificial dynamics	76
3.3.3.1	Preliminary	76
3.3.4	The Two-stage ETKF	79
3.4	Numerical results	81
3.4.1	Generation of the initial ensembles	81
3.4.2	Image deblurring	83
3.4.2.1	Experiment 1: Noise-free blurring model	83
3.4.2.2	Experiment 2: Noise Blurring Model ($\sigma^2 = 0.5$)	86
3.4.2.3	Experiment 3: Noise Blurring Model ($\sigma^2 = 1$)	88
3.4.2.4	Experiment 4: Noise Blurring Model ($\sigma^2 = 1$); Two-stage ETKF	90
3.4.3	Video deblurring	92
3.4.3.1	Numerical result	92
3.5	Summary	93
3.6	Appendix	94
3.6.1	EnKF without perturbation	94
3.6.2	Optimization issue	95
Chapter 4	Conclusion and future work	97
4.1	Conclusion	97
4.2	Future work	98
4.2.1	Discussion on the tensor regression problem	98
4.2.2	Improvement on image restoration by the ensemble method	98
	Bibliography	100
	Curriculum Vitae	105