

DOCTORAL THESIS

Kernel-based least-squares approximations: theories and applications

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Date of Award:
2018

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Abstract

Kernel-based meshless methods for approximating functions and solutions of partial differential equations have many applications in engineering fields. As only scattered data are used, meshless methods using radial basis functions can be extended to complicated geometry and high-dimensional problems. In this thesis, kernel-based least-squares methods will be used to solve several direct and inverse problems.

In chapter 2, we consider discrete least-squares methods using radial basis functions. A general ℓ^2 -Tikhonov regularization with W_2^m -penalty is considered. We provide error estimates that are comparable to kernel-based interpolation in cases in which the function being approximated is within and is outside of the native space of the kernel. These results are extended to the case of noisy data. Numerical demonstrations are provided to verify the theoretical results. In chapter 3, we apply kernel-based collocation methods to elliptic problems with mixed boundary conditions. We propose some weighted least-squares formulations with different weights for the Dirichlet and Neumann boundary collocation terms. Besides fill distance of discrete sets, our weights also depend on three other factors: proportion of the measures of the Dirichlet and Neumann boundaries, dimensionless volume ratios of the boundary and domain, and kernel smoothness. We determine the dependencies of these terms in weights by different numerical tests. Our least-squares formulations can be proved to be convergent at the $H^2(\Omega)$ norm. Numerical experiments in two and three dimensions show that we can obtain desired convergent results under different boundary conditions and different domain shapes. In chapter 4, we use a kernel-based least-squares method to solve ill-posed Cauchy problems for elliptic partial differential equations. We construct stable methods for these inverse problems. Numerical approximations to solutions of elliptic Cauchy problems are formulated as solutions of nonlinear least-squares problems with quadratic inequality constraints. A convergence analysis with respect to noise levels and fill distances of data points is provided, from which a Tikhonov regularization strategy is obtained. A nonlinear algorithm is proposed to obtain stable solutions of the resulting nonlinear problems.

Numerical experiments are provided to verify our convergence results. In the final chapter, we apply meshless methods to the Gierer-Meinhardt activator-inhibitor model. Pattern transitions in irregular domains of the Gierer-Meinhardt model are shown. We propose various parameter settings for different patterns appearing in nature and test these settings on some irregular domains. To further simulate patterns in reality, we construct different kinds of domains and apply proposed parameter settings on different patches of domains found in nature.

Keywords: Meshless, Kansa method, Kernel-based least-squares methods, Cauchy problems, Pattern formations, Gierer-Meinhardt model

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