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Applications of quasi-Monte Carlo methods in model-robust response surface designs

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Preface

Quasi-Monte Carlo methods were originally proposed to solve multidimensional integrals. Compared with a Monte Carlo method, the fundamental principle of a quasi-Monte Carlo method is to generate a deterministic set of points that are uniformly scattered on some region, preferably more uniform than a random sample. The field of quasi-Monte Carlo methods was enriched by the systematic development of the theory of lattice rules and of the theory of nets and (t, s)-sequences. New hybrids of Monte Carlo and quasi-Monte Carlo methods were introduced and some new measures of uniformity were proposed in recent years. Although this is still an active area of research, quasi-Monte Carlo methods have now found applications in other fields, such as computer experiments and uniform designs.

One theme of this thesis is to investigate the sampling variance of integrals estimated by combining two randomized $(\lambda_j, 0, m_j, s)$-nets in base b, $j = 0, 1$, according to Owen (1995). For each individual randomized net, Owen (1997a, 1997b) gives a thorough investigation of the variance. It is interesting to examine the variance for the union of two randomized nets. These two randomizations may or may not be independent. We will give some results for finite sample size and asymptotic situations, respectively.

Another theme of this thesis is to study applications of quasi-Monte Carlo methods in model robust designs. It is known that in classical optimal design theory, the response is described exactly by a particular linear model, and the design is chosen in an optimal manner. However, the assumed model is only an approximation to the true model for the response. Box and Draper (1959) seem to have been the first to take both the variance error and bias error into account in choosing a suitable design which is robust against model bias. More careful investigations on this subject have been done by Kiefer (1973), Huber (1975), Marcus and Sacks (1978), Li (1984), Li and Notz (1982), Notz (1989), Pesotchinsky (1982), Steinberg (1985), and so on. They differ in their choice of the design region, the type of bias, and the loss function to be minimized. In this thesis, it is specified that the class of all possible biases is a reproducing kernel Hilbert space, and the treatments used are worst case analysis and average case analysis. The criteria derived from the bias discrepancy are related to a continuous uniform distribution. Therefore, it is interesting to consider performances of quasi-Monte Carlo methods under the criteria for model robust designs.

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