

## DOCTORAL THESIS

### Teachers' perceptions of Ill-posed mathematical problems: implications of task design for implementation of formative assessments

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Abstract of this thesis entitled

**“Teachers’ Perceptions of Ill-posed Mathematical Problems:  
Implications of Task Design  
for Implementation of Formative Assessments”**

Submitted by

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By manipulating constraints and goals, this study had generated some ill-posed problems in “Fractions” which were packed into 2 mathematical tasks for teacher uses in an intended exploration of their perceived effectiveness of teaching mathematical problem-solving against their student responses through the lens of the theory of formative assessment. Each ill-posed problem was characterized by certain descriptive “instability” that users would have to define own sets of mathematical assumptions for problem-solving inquiries. 3 highly qualified, experienced, and trained mathematics teachers were purposefully recruited, and instructed to acquire and mark student responses without any prior teaching and intervention. Each of these teachers’ perceptions of ill-posed problems was acquired through a semi-structured clinical case-interview. All teachers in common demonstrated only individual singular mathematical problem-solving inquiries as major instructional adjustments during evaluation, even though individuals had ample opportunities in manipulating the described intention of each problem. Although some could realize inquiries from students being alternative to own used, not all would intend to change initial instructional plans of each problem and could design dedicated tasks in extending given problem-solving contexts for subsequent teaching and maintaining the described problem-solving intentions merely because of evaluation purposes. The resulting thick teacher perceptions were then analyzed by the Mayring’s (2015) Qualitative Content Analysis (QCA) method for exploring particularly those who could intend to influence and get influenced by students’ used mathematical assumptions in interviews. Certain unanticipated uses of assumptions of student individuals and groups were evidently found to have influenced cognitively some teachers’ further problem-solving inquiries at some interview instants and stimulated their perception changes. In the lack of subject implementation in mathematics education for the theory of “formative” assessment (Black & Wiliam, 2009), based on its definition, these instants should be put as their potential creations of and/or capitalizations upon certain asynchronous moments of contingency according to their planning of instructional

adjustments for more comprehensive learning and definite growths of mathematical inquiries of students according to individuals' needs of problem-solving. Due to QCA, these perception changes might be characterized by four certain inductively formed categories of scenarios of perceptions, which were summarized as 1) Evaluation Perception, 2) Assumption Expansion Perception, 3) Assumption Collection Perception, and 4) Intention Indecision Perception. These scenarios of perceptions might be used to explore teachers' intentions, actions, and coherency in accounting for students' used assumptions in mathematical inquiries for given problem-solving contexts and extensions of given intentions of mathematical inquiries, particularly in their designs of mathematical tasks. Teacher uses of ill-posed problems were shown to have provided certain evidences in implementing formative assessments which should substantiate a subject implementation of its theory in the discipline of mathematics education. Methodologically, the current study also substantiate how theory-guided designs of ill-posed problems as well as generic plain text analysis through QCA have facilitate effectiveness comparisons of instructional adjustments within a teacher, across different teachers, decided prior knowledge, students of prior mathematical learning experiences, and students in different levels of schooling and class size.

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APPENDIX

A

A Sample of Teacher's STUDY INFORMATION SHEET  
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