Intrinsic meshless methods for PDEs on manifolds and applications

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Abstract

Radial basis function (RBF) methods for partial differential equations (PDEs), either in bulk domains, on surfaces, or in a combination of the formers, arise in a wide range of practical applications. This thesis proposes numerical approaches of RBF-based meshless techniques to solve these three kinds of PDEs on stationary and nonstationary surfaces and domains.

In Chapter 1, we introduce the background of RBF methods, some basic concepts, and error estimates for RBF interpolation. We then provide some preliminaries for manifolds, restricted RBFs on manifolds, and some convergence properties of RBF interpolation. Finally, implicit-explicit time stepping schemes are briefly presented.

In Chapter 2, we propose methods to implement meshless collocation approaches intrinsically to solve elliptic PDEs on smooth, closed, connected, and complete Riemannian manifolds with arbitrary codimensions. Our methods are based on strong-form collocations with oversampling and least-squares minimizations, which can be implemented either analytically or approximately. By restricting global kernels to the manifold, our methods resemble their easy-to-implement domain-type analogies, that is, Kansa methods. Our main theoretical contribution is a robust convergence analysis under some standard smoothness assumptions for high-order convergence. We simulate reaction-diffusion equations to generate Turing patterns and solve shallow water problems on manifolds.

In Chapter 3, we consider convective-diffusion problems that model surfactants or heat transport along moving surfaces. We propose two time-space algorithms by combining the methods of lines and kernel-based meshless collocation techniques intrinsic to surfaces. We use a low-order time discretization for fair comparison, and higher-order schemes in time are possible. The proposed methods can achieve second-order convergence. They use either analytic or approximated spatial discretization of the surface operators, which do not require regeneration of point clouds at each temporal iteration. Thus, they are alternatively applied to handle models on two types of evolving surfaces, which are defined as prescribed motions and governed by geometric evolution laws, respectively. We present numerical examples on various evolving surfaces for the performance of our algorithms and apply the approximated one to merging surfaces.

In Chapter 4, a kernel-based meshless method is developed to solve coupled second-order elliptic PDEs in bulk domains and on surfaces, subject to Robin boundary conditions. It combines a least-squares kernel-based collocation method with a surface-type intrinsic approach. We can thus use each pair for discrete point sets, RBF kernels (globally and restrictedly), trial spaces, and some essential assumptions, to search for least-squares solutions in bulks and on surfaces, respectively. We first analyze error estimates for a domain-type Robin-boundary problem. Based on this analysis and the existing results for surface PDEs, we discuss the theoretical requirements for the Sobolev kernels used. We then select the orders of smoothness for the kernels in bulks and on surfaces. Finally, several numerical experiments are demonstrated to test the robustness of the coupled method in terms of accuracy and convergence rates under different settings.

Keywords: intrinsic kernel-based meshless collocation methods, radial basis functions, convergence analysis, least-squares minimizations, evolving and merging surfaces, parabolic PDEs, coupled bulk-surface PDEs.

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