

DOCTORAL THESIS

Intrinsic meshless methods for PDEs on manifolds and applications

Chen, Meng

Date of Award:
2018

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

Abstract

Radial basis function (RBF) methods for partial differential equations (PDEs), either in bulk domains, on surfaces, or in a combination of the formers, arise in a wide range of practical applications. This thesis proposes numerical approaches of RBF-based meshless techniques to solve these three kinds of PDEs on stationary and nonstationary surfaces and domains.

In Chapter 1, we introduce the background of RBF methods, some basic concepts, and error estimates for RBF interpolation. We then provide some preliminaries for manifolds, restricted RBFs on manifolds, and some convergence properties of RBF interpolation. Finally, implicit-explicit time stepping schemes are briefly presented.

In Chapter 2, we propose methods to implement meshless collocation approaches intrinsically to solve elliptic PDEs on smooth, closed, connected, and complete Riemannian manifolds with arbitrary codimensions. Our methods are based on strong-form collocations with oversampling and least-squares minimizations, which can be implemented either analytically or approximately. By restricting global kernels to the manifold, our methods resemble their easy-to-implement domain-type analogies, that is, Kansa methods. Our main theoretical contribution is a robust convergence analysis under some standard smoothness assumptions for high-order convergence. We simulate reaction-diffusion equations to generate Turing patterns and solve shallow water problems on manifolds.

In Chapter 3, we consider convective-diffusion problems that model surfactants or heat transport along moving surfaces. We propose two time-space algorithms by combining the methods of lines and kernel-based meshless collocation techniques intrinsic to surfaces. We use a low-order time discretization for fair comparison, and higher-order schemes in time are possible. The proposed methods can achieve second-order convergence. They use either analytic or approximated spatial discretization of the surface operators, which do not require regeneration of point clouds at each temporal iteration. Thus, they are alternatively applied to handle models on two types of evolving surfaces, which are defined as prescribed motions and governed by

geometric evolution laws, respectively. We present numerical examples on various evolving surfaces for the performance of our algorithms and apply the approximated one to merging surfaces.

In Chapter 4, a kernel-based meshless method is developed to solve coupled second-order elliptic PDEs in bulk domains and on surfaces, subject to Robin boundary conditions. It combines a least-squares kernel-based collocation method with a surface-type intrinsic approach. We can thus use each pair for discrete point sets, RBF kernels (globally and restrictedly), trial spaces, and some essential assumptions, to search for least-squares solutions in bulks and on surfaces, respectively. We first analyze error estimates for a domain-type Robin-boundary problem. Based on this analysis and the existing results for surface PDEs, we discuss the theoretical requirements for the Sobolev kernels used. We then select the orders of smoothness for the kernels in bulks and on surfaces. Finally, several numerical experiments are demonstrated to test the robustness of the coupled method in terms of accuracy and convergence rates under different settings.

Keywords: intrinsic kernel-based meshless collocation methods, radial basis functions, convergence analysis, least-squares minimizations, evolving and merging surfaces, parabolic PDEs, coupled bulk-surface PDEs.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iv
Table of Contents	v
Chapter 1 Background and preliminaries	1
1.1 Radial basis function methods	1
1.1.1 RBFs	2
1.1.2 Native spaces	4
1.1.3 Error estimates for RBF interpolation	6
1.2 RBFs restricted on manifolds	7
1.2.1 Manifolds	7
1.2.2 Sobolev spaces on manifolds	8
1.2.3 Restricted RBFs and error estimates	10
1.3 Time stepping	11
1.4 Outline of Thesis	13
Chapter 2 Intrinsic Kansa-type meshless methods for PDEs on manifolds	15
2.1 Introduction	15
2.2 Sobolev spaces on manifolds and embedding domains	17
2.3 Partial differential equations on surfaces	19
2.3.1 Kernels and discrete settings	20

2.3.2	Stability and consistency	22
2.4	Intrinsic meshless collocation methods and implementation	26
2.4.1	Kansa-type projection method	26
2.4.2	Approximated Kansa method	28
2.5	Numerical demonstrations	29
2.5.1	Effects of oversampling	30
2.5.2	Smoothness of kernels	32
2.5.3	Pattern formations and robustness	33
2.5.4	Shallow water problems on surfaces	36
2.6	Conclusions	39
2.7	Future works	39
2.8	Appendix	42
2.8.1	Parametric equations for all tested manifolds	42
2.8.2	Initial conditions of shallow water equations on surfaces	43
Chapter 3 Intrinsic meshless collocation methods for solving PDEs on evolving and merging surfaces		44
3.1	Introduction	44
3.2	Parameterized surfaces with prescribed evolution and an algorithm for convection-diffusion equations	47
3.2.1	Semi-discretization in time	48
3.2.2	Algorithm for solving PDEs on prescribed moving surfaces	50
3.3	Moving surfaces based on point clouds	52
3.3.1	Geometric evolution of surfaces	53
3.3.2	Numerical solver based on point clouds	54
3.4	Numerical examples	57
3.4.1	Prescribed evolving surfaces	57
3.4.2	Mass conservation on prescribed surfaces and simulations on geometric evolving surfaces	63
3.5	Conclusions	67

3.6	Future works	68
Chapter 4 Kernel-based meshless collocation methods for solving coupled bulk-		
	surface PDEs	73
4.1	Introduction	73
4.2	Coupled bulk-surface elliptic PDEs	74
4.3	Discrete settings, kernels and trial spaces	75
4.3.1	Discrete settings	75
4.3.2	Kernels	75
4.3.3	Bulk and surface trial spaces	76
4.4	Convergence analysis for bulk PDEs with Robin boundary conditions	77
4.4.1	Essential inequalities	78
4.4.2	Convergence results for bulk PDEs	81
4.5	Meshless collocation methods for coupled bulk-surface PDEs	83
4.5.1	Implementation	83
4.5.2	Some discussions on convergence	84
4.6	Numerical experiments	87
4.6.1	Smoothness orders of kernels for accuracy and convergence . .	88
4.6.2	Point settings for bulk and surface in a torus	93
4.6.3	Simulations on other surfaces	95
4.7	Conclusions	97
4.8	Future works	98
4.8.1	Error analysis	98
4.8.2	Other problems	99
	Bibliography	101
	Curriculum Vitae	111