DOCTORAL THESIS

Theoretical advances on scattering theory, fractional operators and their inverse problems

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Abstract

Inverse problems arise in numerous fields of science and engineering where one tries to find out the desired information of an unknown object or the cause of an observed effect. They are of fundamental importance in many areas including radar and sonar applications, nondestructive testing, image processing, medical imaging, remote sensing, geophysics and astronomy among others. This study is concerned with three issues in scattering theory, fractional operators, as well as some of their inverse problems.

The first topic is scattering problems for electromagnetic waves governed by Maxwell equations. It will be proved in the current study that an inhomogeneous EM medium with a corner on its support always scatters by assuming certain regularity and admissible conditions. This result implies that one cannot achieve invisibility for such materials. In order to verify the result, an integral of solutions to certain interior transmission problem is to be analyzed, and complex geometry optics solutions to corresponding Maxwell equations with higher order estimate for the residual will be constructed.

The second problem involves the linearized elastic or seismic wave scattering described by the Lamé system. We will consider the elastic or seismic body wave which is composed of two different type of sub-waves, that is, the compressional or primary (P-) and the shear or secondary (S-) waves. We shall prove that the P- and the S-components of the total wave can be completely decoupled under certain geometric and boundary conditions. This is a surprising finding since it is known that the P- and the S-components of the elastic or seismic body wave are coupled in general. Results for decoupling around local boundary pieces, for boundary value problems, and for scattering problems are to be established. This decoupling property will be further applied to derive uniqueness and stability for the associated inverse problem of identifying polyhedral elastic obstacles by an optimal number of scattering measurements.

Lastly, we consider a type of fractional (and nonlocal) elliptic operators and the associated Calderón problem. The well-posedness for a kind of forward problems concerning the fractional operator will be established. As a consequence, the corresponding Dirichlet to Neumann map with certain mapping property is to be defined. As for the inverse problem, it will be shown that a potential can be uniquely identified by local Cauchy data of the associated nonlocal operator, in dimensions larger than or equal to two.

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