

DOCTORAL THESIS

Distance two labeling of some products of graphs

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Distance Two Labeling of Some Products of Graphs

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Abstract

Let j , k and m be positive numbers, an $L(j, k)$ -labeling f of G is an assignment of numbers to vertices of G such that $|f(u) - f(v)| \geq j$ if $d(u, v) = 1$, and $|f(u) - f(v)| \geq k$ if $d(u, v) = 2$. Then the *span* of f is the difference between the maximum and the minimum numbers assigned by f . The $L(j, k)$ -number of G , denoted by $\lambda_{j,k}(G)$, is the minimum span over all $L(j, k)$ -labelings of G .

A *circular m - $L(j, k)$ -labeling* of a graph G is a function $f : V(G) \rightarrow [0, m)$ such that $|f(u) - f(v)|_m \geq j$ if u and v are adjacent, and $|f(u) - f(v)|_m \geq k$ if u and v are at distance two, where $|a - b|_m = \min\{|a - b|, m - |a - b|\}$. The minimum m such that there exist a circular m - $L(j, k)$ -labeling of G is called the *circular- $L(j, k)$ -number* of G and is denoted by $\sigma_{j,k}(G)$.

In this thesis, for any two positive numbers j and k with $2j \leq k$, we determine the $L(j, k)$ -numbers and circular $L(j, k)$ -numbers of the direct product of a path and a cycle. Moreover, we also investigate the $L(j, k)$ -numbers of the Cartesian product of a path and a cycle.

Keywords: Graph, $L(j, k)$ -labeling, circular- $L(j, k)$ -labeling, $L(j, k)$ -number, circular- $L(j, k)$ -number, direct product, Cartesian product.

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