

## MASTER'S THESIS

### Globally convergent and efficient methods for unconstrained discrete-time optimal control

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**Globally Convergent and Efficient Methods for  
Unconstrained Discrete-Time Optimal Control**

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## Abstract

In solving unconstrained discrete-time optimal control problems by the differential dynamic programming (DDP) method, shift schemes are commonly used to treat non-convex situations. However, the existing shift schemes are inefficient when the shift is too large. In this dissertation, a new method of combining the DDP method with the automatic shift scheme and the steepest descent method is proposed to cope with these situations. Under the standard assumptions, the proposing method is globally convergent and has q-quadratic local convergence. Extensive numerical experiments on several test problems in the literature are reported. These numerical results indicate that the proposing method is robust and very efficient.

Similarly, the same technique is applied to the quasi-Newton differential dynamic programming (QDDP) method. Under the standard assumptions, the proposing method is also globally convergent. Since the efficiency of the QDDP method depends heavily on the updating of the quasi-Newton approximations to the second order partial derivatives in the stagewise quasi-Newton condition, five secant updates are compared. Extensive numerical experiments are reported. These numerical results indicate that the BFGS update is the best among the five secant updates on average. Furthermore, the proposing method with the BFGS update can improve the robustness and efficiency of the QDDP method.

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