

MASTER'S THESIS

Globally convergent and efficient methods for unconstrained discrete-time optimal control

Ng, Chi Kong

Date of Award:
1998

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

**Globally Convergent and Efficient Methods for
Unconstrained Discrete-Time Optimal Control**

NG Chi Kong

A thesis submitted in partial fulfilment of the requirements

for the degree of

Master of Philosophy

October 1998

Hong Kong Baptist University

Abstract

In solving unconstrained discrete-time optimal control problems by the differential dynamic programming (DDP) method, shift schemes are commonly used to treat non-convex situations. However, the existing shift schemes are inefficient when the shift is too large. In this dissertation, a new method of combining the DDP method with the automatic shift scheme and the steepest descent method is proposed to cope with these situations. Under the standard assumptions, the proposing method is globally convergent and has q-quadratic local convergence. Extensive numerical experiments on several test problems in the literature are reported. These numerical results indicate that the proposing method is robust and very efficient.

Similarly, the same technique is applied to the quasi-Newton differential dynamic programming (QDDP) method. Under the standard assumptions, the proposing method is also globally convergent. Since the efficiency of the QDDP method depends heavily on the updating of the quasi-Newton approximations to the second order partial derivatives in the stagewise quasi-Newton condition, five secant updates are compared. Extensive numerical experiments are reported. These numerical results indicate that the BFGS update is the best among the five secant updates on average. Furthermore, the proposing method with the BFGS update can improve the robustness and efficiency of the QDDP method.

Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Contents	iv
List of Tables	viii
1 Introduction	1
1.1 Unconstrained Discrete-Time Optimal Control Problems (UDOCPs)	1
1.2 The Dynamic Programming (DP) Method for UDOCPs	2
1.3 Notations	4
1.4 Organization of the Dissertation	6
2 The Differential Dynamic Programming (DDP) Method for UDOCPs	8
2.1 The Locally Convergent DDP Method for Convex UDOCPs	8
2.1.1 The Backward Sweep	9

2.1.2	The Forward Sweep	12
2.1.3	Local Convergence	12
2.2	The Globally Convergent Modifications of the DDP Method	13
2.2.1	The Backward Sweep with the Automatic Shift Scheme	13
2.2.2	The Forward Sweep with a Line Search Scheme	16
2.2.3	Global Convergence and Local Convergence	18
3	The Steepest Descent (SD) Method for UDOCPs	20
3.1	The Backward Sweep	20
3.2	The Forward Sweep with a Line Search Scheme	22
3.3	Convergence Analysis	27
4	A New Efficient Method that Combines Both the DDP and SD Methods for UDOCPs	28
4.1	The Motivation	28
4.2	The Algorithm	29
4.3	Convergence Analysis	30
4.4	Numerical Experiment	34
5	The Quasi-Newton Differential Dynamic Programming (QDDP) Method for UDOCPs	42
5.1	The Locally Convergent QDDP Method for Convex UDOCPs	42
5.1.1	The Backward Sweep	43
5.1.2	The Forward Sweep	45
5.1.3	Local Convergence	46

5.2	Globally Convergent Modifications of the QDDP Method	46
5.2.1	The Backward Sweep with the Automatic Shift Scheme . . .	47
5.2.2	The Forward Sweep with a Line Search Scheme	49
5.2.3	Global Convergence	51
5.3	The Stagewise Quasi-Newton (SQN) Conditions in the QDDP Method	54
6	A New Efficient Method that Combines Both the QDDP and SD	
	Methods for UDOCPs	59
6.1	The Algorithm	59
6.2	Convergence Analysis	60
6.3	Numerical Experiment	62
7	Conclusions and Further Developments	73
7.1	Conclusions	73
7.2	Further Developments	75
	References	76
	A Test Problems	78
	B Numerical Results of the DDP Method with $N+1=10$	82
	C Numerical Results of the DDP Method with $N+1=30$	87
	D Numerical Results of the QDDP Method with $N+1=10$	92
	E Numerical Results of the QDDP method with $N+1=30$	101

