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DOCTORAL THESIS

Spectral collocation methods for the fractional PDEs in unbounded domain

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Abstract

This thesis is concerned with a particular numerical approach for solving the fractional partial differential equations (PDEs). In the last two decades, it has been observed that many practical systems are more accurately described by fractional differential equations (FDEs) rather than the traditional differential equation approaches. Consequently, it has become an important research area to study the theoretical and numerical aspects of various types of FDEs. This thesis will explore high order numerical methods for solving FDEs numerically. More precisely, spectral methods which exhibits exponential order of accuracy will be investigated. The method consists of expanding the solution with proper global basis functions and imposing collocation conditions on the Gauss quadrature points.

In this work, Hermite and modified rational functions are employed to serve as basis functions for solutions that decay exponentially and algebraically, respectively. The main emphasis of this thesis is to propose the spectral collocation method for FDEs posed in *unbounded domains*. Components of the differentiation matrix involving fractional Laplacian are derived which can then be computed recursively using the properties of confluent hypergeometric function or hypergeometric function.

The first part of the thesis introduces preliminaries useful for other parts of the thesis. Review of the relevant definitions and properties of special functions such as Hermite functions, Bessel functions, hypergeometric functions, Gegenbauer polynomials, mapped Jacobi polynomials, modified rational functions are presented. Fractional Sobolev space is introduced and some lemmas on interpolation approximation in the fractional Sobolev space are also included.

In the second part of the thesis, we present the spectral collocation method based on Hermite functions. Two bases are used, namely, the over-scaled Hermite function and generalized Hermite function, which are orthogonal functions on the whole line with appropriate weight functions. We will show that the fractional Laplacian of these two kinds of Hermite functions can be represented by confluent hypergeometric function. Behaviors of the condition numbers for the resulting spectral differentiation matrices with respect to the number of expansion terms are investigated.

Moreover, approximation in two-dimensional space using the tensorized bases, application to multi-term problems and use of scaling to match different decay rate are also considered. Convergence analysis for generalized Hermite function are derived and numerical errors for two bases are analyzed.

The third part of the thesis deals with the spectral collocation method based on modified rational functions. We first give a brief introduction for computation of the fractional Laplacian using modified rational functions, which is represented by hypergeometric functions. Then the differentiation matrix involving the fractional Laplace operator is given. Convergence analysis for modified Chebyshev rational functions and modified Legendre rational functions are derived and numerical experiments are carried out.

Keywords: Fractional PDEs, Hermite polynomials/functions, Gegenbauer polynomials, mapped Jacobi functions, modified rational function, unbounded domain, spectral collocation methods.

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