DOCTORAL THESIS

Adaptive meshless methods for solving partial differential equations

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Adaptive Meshless Methods for Solving Partial Differential Equations

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ABSTRACT

RBF collocation method is one of the most popular meshless computational method and many applications of RBF collocation method in different areas can be found. In this thesis, we discuss the RBF collocation method for solving partial differential equations and related issues. Since some non-stationary problems can be treated as stationary problems after discretizing the time derivative, we consider the stationary problems first. We give the convergence proof of a least-squares asymmetric radial basis function collocation method for solving the modified Helmholtz equations which is the reduced problem of some non-stationary problems such as heat equation. Second, we propose a method to enhance the performance for solving 3D inhomogeneous elliptic equations. The above methods show the excellent performance if the solution is smooth enough. Third, instead of uniform nodes, refinement nodes are used if the solution contains sharp region. We show that using refinement nodes results in more accurate RBF approximation than uniform nodes if the solution contains high variations. Besides, the Hybrid RBF approach will be discussed for meshless interpolation and approximation. Lastly, we combine all developed methods to give an adaptive method for solving time-dependent partial differential equations. Some numerical examples of the heat and the Burger's equations will be given to conclude the work.

Table of Contents

Declaration i

Abstract ii

Acknowledgement iii

Table of Contents iv

List of Figures vii

Chapter 1 Introduction 1

Chapter 2 On convergence of a least-squares Kansa's method for the modified Helmholtz equations 6

2.1 Introduction 6

2.2 Overdetermined least-squares Kansa's method 8

2.3 Convergence analysis 10

2.3.1 RBF interpolation 10

2.3.2 Continuous dependence on data 11

2.3.3 The Denseness requirement for meshless collocation 13

2.3.4 Error estimates 16

2.4 Numerical demonstrations of exponential convergence 17

2.4.1 Example I (1D-HP) 18

2.4.2 Example II (2D-HP and DP) 19

2.4.3 Example III (3D-DP-Greedy) 21

2.5 Conclusion 22

Chapter 3 Solving 3D inhomogeneous elliptic equations adaptively with the method of particular solutions 25
3.1 Introduction 25
3.2 One–stage method of particular solutions 27
3.3 Methodology 29
3.3.1 Notations 30
3.3.2 Adaptive greedy algorithm 31
3.4 Numerical examples 32
3.5 Conclusions 35

Chapter 4 Moving data points method 39
4.1 Introduction 39
4.2 RBF interpolation 40
4.3 Moving data points method 41
4.3.1 One-dimensional equidistribution principle 41
4.3.2 Dimension splitting scheme 42
4.3.3 Polynomial curve averaging 44
4.4 Numerical experiments 46
4.4.1 Numerical Examples of moving data point method 47
4.4.2 Numerical examples for meshless approximation 48
4.5 Conclusion 55

Chapter 5 Coupling adaptive node refinement and hybrid RBF approach for meshless approximation 58
5.1 Introduction 58
5.2 Rules for splitting cells 59
5.2.1 Interior nodes refinement 59
5.2.2 Near–Boundary nodes refinement 60
5.2.3 Quad-tree procedure 61
5.3 Adaptive greedy algorithm 62
5.4 Hybrid adaptive global–local RBF approach 62
5.5 Numerical demonstrations 65

5.5.1 Numerical demonstrations for splitting cells 65
5.5.2 Numerical demonstrations for adaptive node with hybrid adaptive global–local RBF solver 69
5.6 Conclusion 77

Chapter 6 The extension to Time-dependent partial differential equations 78
6.1 Introduction 78
6.2 Methodology 79
6.3 Numerical examples of Heat equation 80
6.3.1 Heat equation with periodic solution 80
6.3.2 Heat equation with decaying solution 82
6.3.3 Heat equation with non-smooth solution 83
6.4 Numerical example of Burgers’ equation 83
6.5 Conclusion 85

Chapter 7 Conclusions 91

Bibliography 93

Curriculum vitae 98