

MASTER'S THESIS

Moving mesh methods for viscoelastic flows with free boundaries

Zhang, Yubo

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Moving Mesh Methods for Viscoelastic Flows with Free Boundaries

ZHANG Yubo

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Abstract

The application of harmonic map based moving mesh methods for free boundary viscoelastic flows is studied both in 2D and 3D cases. In 2D case, we adopt the phase-field model and finite element discretization to solve two-phase viscoelastic flows. Since the complexity and scale of the problem grows fast in 3D, a novel and efficient finite difference framework on moving meshes is developed for the free surface viscoelastic flows and the level set calculations. In the new framework we designed a logical domain semi-Lagrangian scheme for moving mesh solution interpolation and convection. Numerical results show that harmonic map based moving mesh methods can achieve better accuracy for viscoelastic flows with free boundaries while using much less memory and computational time compared to the uniform mesh simulations. Some numerical simulation results also agreed well with physical experiments.

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