DOCTORAL THESIS

Local absorbing boundary conditions for some nonlinear PDEs on unbounded domains

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Local Absorbing Boundary Conditions for Some Nonlinear PDEs on Unbounded Domains

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Abstract

Many physical phenomena are modeled by the time-dependent PDEs on unbounded domain such as heat equation, Schrödinger-type equations, and semilinear parabolic equations with blow-up solution, nonlinear Schrödinger equation, sine-Gordon equation and so on. This thesis is concerned with the numerical solution of this kind of PDEs on unbounded spatial domain. The focus of the presentation is on the derivation of absorbing boundary conditions for one-dimensional and two-dimensional domains, which can be extended to high-dimensions.

One of the main contributions of the thesis is devoted to the high-order local absorbing boundary conditions (ABCs) for heat and Schrödinger equations. We proved that the coupled system yields a well-posed problem between the obtained higher-order local ABCs and the partial differential equation on the bounded computational domain. This method has been used widely in wave propagation models only recently. We extend the spirit of the methodology to parabolic ones, which will become a basis to design the local ABCs for nonlinear models. Some numerical tests show that the new treatment is very efficient and tractable.

Another main contribution of the thesis is to propose a unified approach to the design of local absorbing boundary conditions for some nonlinear problems. Based on the time-splitting idea, the main process of the unified approach is to approximate the linear term by a one-directional equation, combine the results with the nonlinear term, and then obtain the well-posed and accurate local ABCs on the artificial boundaries. For multi-dimensional case, we use the (1,1)-Padé approximation to the linear term, and also unite it with the nonlinear term to give some local absorbing boundary conditions at corners. Thus we propose the correspond local ABCs for semilinear parabolic equation with blow-up solution and nonlinear Schrödinger equations on one-dimensional and higher-dimensional domains. Numerical examples are given to illustrate the stable and tractable advantages of the method.

Finally, we use an analytical-numerical approach to find, in a systematic way, new 1-soliton solutions for a discrete sine-Gordon system in one spatial dimension. Since the spatial domain is unbounded, the numerical scheme employed to generate these soliton solutions is based on the artificial boundary method. A large selection of numerical examples provides much insight into the possible shapes of these new 1-solitons.

Keywords: Local absorbing boundary conditions, unified approach, Semilinear PDEs, unbounded spatial domains, finite-time blow-up, finite difference spatial discretization, adaptive time-stepping, sine-Gordon equation, Schrödinger-type equations, numerical soliton solutions, sine-Gordon equation.

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