

## MASTER'S THESIS

### Dimension reduction in the regressions through weighted variance estimation

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# Dimension Reduction in Regressions through Weighted Variance Estimation

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# Abstract

Sliced average variance estimation (SAVE) is an important dimension reduction method in the sufficient dimension reduction area. However, its efficacy relies heavily on the choice of the number of slices. In this thesis we introduce a class of weighted variance estimation (WVE), which, similar to SAVE and simple contour regression (SCR), uses the conditional variance of the inverse regression to recover the central subspace. In chapter 1 and chapter 2 we will briefly review some concepts about dimension reduction and other methods. In chapter 3, the population properties of WVE are studied, and their connection with classical dimension reduction methods is also established. To extend the application of WVE to regressions with diverging number of predictors, the strong consistency and asymptotic normality of the kernel estimation of WVE are established under some mild regularity conditions. In the last chapter, finite sample studies are carried out for comparison with the existing methods and a reaction data example is presented for illustration.

**Keywords:** Central Mean Subspace; Central Subspace; dimension reduction; Sliced Average Variance Estimator; Sliced Inverse Regression; Asymptotic normality; Diverging parameters; High dimensionality.

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