# DOCTORAL THESIS

## On single-index model and its related topics

Chang, Ziqing

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# On Single-index Model and Its Related Topics

**CHANG Ziqing**

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Principal Supervisor: Professor Lixing Zhu

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# Abstract

Parametric regression modeling is popularly used technique in regression analysis. The single-index model is a mostly investigated semi-parametric model for several decades. It maintains many of the desirable features of linear model and least-squares methods. Several methods, from direct derivative estimation method, the simultaneous minimization method to dimension reduction methods, have been provided by researchers. Generally, we are interested in the estimation of the index parametric vector, while viewing the unknown link function as an infinite dimensional nuisance parameters. The desired objective is to obtain a root-$n$ consistent estimator of the index parameter and an optimal smoothing fitting of the link function.

In this thesis, we first provide a review of the estimation and inference of the single-index model and smoothing spline model and its hypothesis testing. Then we consider the estimating equation for the single-index model, and give an estimator with smaller variance than those in existing literatures as well as an optimal smoothing of the link function. The asymptotic properties of the estimators are provided and the results of our two step estimation show that there exists more effective estimator. This phenomenon of estimation could also take place in a regular semiparametric estimating procedure. Simulated studies conduct the performances of bias, variance and minimum squared error of two step estimator as well as the comparisons with sliced inverse regression and least squared optimization. We also employ a general way to obtain the score inference of this model. Two distinct semiparametric estimating equations (generally called as adaptive equation and effective equation) of the single-index model have been considered under regularity conditions for local linear smoothing model. Asymptotic properties of estimators and Wald statistics are provided in more general expressions. Simulation studies perform the comparisons of these estimators, and 0.95, 0.9, 0.5 confidence regions of the index parameters are conducted by Wald statistics.

Secondly, we consider a topic of hypothesis testing of nonparametric regression how depart a nonparametric regression to a parametric regression with a linear regression structure, which is almost in the meaning of series approximation. Following the spirit of Grace Wahba, we conduct the generalized U statistics for nonparametric regression under smoothing spline modeling. Asymptotic properties of our statistics are provided and simulation studies are performed comparisons with existent methods. This goodness-of-fit test of nonparametric regression could provide a way to detect the reduction of single-index model into a linear regression or other complex linear regression structures. However, such a modeling needs a model checking to be accompanied. On the other hand, to keep flexibility and simplicity of modeling, the single-index modeling attracts much attention.

At last, we construct the empirical likelihood of local linear smoothing by a matrix transform of general estimating equation of local linear smoothing. The asymptotic results of maximum empirical likelihood estimator are corresponding to local linear smoothing. The inference methods of nonparametric regression play the key role of the hypothesis topics of single-index model. Thus, the goodness-of-fit test of smoothing spline and empirical likelihood modeling of local linear smoothers provide the methods preparation of further research of single-index models.

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