

## DOCTORAL THESIS

### Image denoising using wavelet domain hidden Markov models

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# Image Denoising Using Wavelet Domain Hidden Markov Models

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# Abstract

Most recently, wavelet domain hidden Markov models (WD-HMMs) are proposed for image processing. The basic idea of WD-HMM is that the dependencies among wavelet coefficients can be efficiently captured by hidden Markov models (HMMs) since the dependency between two wavelet coefficients dies down quickly as their distance becomes big. Besides this, WD-HMM also allows us to approximate the distribution of the wavelet coefficient with a Gaussian Mixture model (GMM). Since WD-HMMs construct the HMMs on the wavelet domain, they lead the model simpler than the traditional statistical models in the spatial domain.

However, most of the improvements of the WD-HMM focus on how to add some additional structures on the original WD-HMM. Generally speaking, the new structures will improve the performance of image processing, however, even some simple additional structures will lead to increasing computational complexity significantly.

Image denoising without blurring the edges is a difficult problem. Constructing adaptable denoising algorithms is an important technique to solve this problem. However, most of spatial denoising techniques blur the edges and singularities of the images since they consider the details of the images are the same as the noise. In this thesis, we firstly propose a new adaptive denoising framework on wavelet domain and then extend it to WD-HMMs with some new local structures.

The new adaptive denoising techniques based on the fact that the images are non-stationary with singularities and some smooth areas, which can be considered as stationary. Firstly, the singularities are separated from the smooth areas. Thus we can handle the different coefficients separately. The local squares is defined on the context, that is, the variance of the singularity will be estimated in relatively a large square while the variance of the smooth

coefficients will be estimated in a smaller square. this new framework is different from traditional local context methods, which estimate the variance of the signal using the pixels with the same context in a moving windows. In order to reduce the artifacts in the denoising images, we construct a template in LL subband and then used it to all subbands. This new technique combing with the block model will be extended to WD-HMMs.

Our new frameworks consider each subband of the wavelet coefficients to be a Gaussian mixture field (GMF), that is, each wavelet coefficient is a random variable with GMM, and allows the dependency links among the hidden states of the wavelet coefficients. Therefore, the joint distribution of each subband can be easily decided by the new frameworks. Then the standard parameter estimation of the new models can be obtained from the EM algorithm and the estimated parameters are used for signal and image denoising.

In order to obtain the adaptable image denoising results, we must obtain the local estimated parameters firstly. Based on carefully designed local structure on wavelet domain, we can use the same local squares and further consider the block structure which coincides the non-stationary of images. That is, in the local squares, we will consider not only the number of coefficients with different context but also consider the block labels of these coefficients. This help us to correct the local structure of the local denoising technique. Thus the estimation will be in the same adaptive squares and blocks. We know that the template can be used to reduce the artifacts in the denoising images. In fact, the template also can help us properly construct the adaptive structure in noisy. This will be discussed in our thesis.

After obtaining the estimated parameters on the wavelet domain, we can use these parameters for image denoising on the wavelet domain. Finally, the denoised image can be obtained from an inverse wavelet transform.

We give some examples on signal and image denoising using the block HMM and the template HMM relatively to show the power and potentiality of the new frameworks. The experimental results show that the block HMM and the template HMM can efficiently improve the spatial adaptability in a simple way.

They also show that signals with relatively stable nature and images with a proper structure of the texture have better denoising results. Finally, we give the summary of our works and discuss the future work.

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