DOCTORAL THESIS

# Construction of wavelets based on unitary transform, permutation and matrix extension with applications to watermarking

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# Construction of Wavelets Based on Unitary Transform, Permutation and Matrix Extension with Applications to Watermarking

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# Abstract

Since 1980s, wavelet analysis has been a popular field in scientific research. To apply wavelet methods to digital image processing, two-dimensional wavelets have to be constructed. However, there still exists many open problems for the construction of multidimensional wavelets, and only some concrete results have been obtained for certain special two-dimensional examples. On the other hand, non-separable wavelets have attractive properties for some applications such as compression, watermarking, etc. In this thesis, we are mainly concerned with the construction of two-dimensional wavelets and their applications in watermarking. Several significant results have been obtained. Some of these results are also well suited to one-dimensional wavelets.

The unitary transform of conjugate quadrature filter (CQF) is proposed. By unitary transform of one-dimensional CQF, we provide a parameterization of 2-band one-dimensional orthogonal wavelet filters. Any 2-band one-dimensional orthogonal wavelet filters can be given explicitly. By unitary transformation of matrix CQF, we provide an algorithm for constructing orthogonal multiwavelets from the corresponding multiscaling function. Applying this transformation once, the orders of the polynomials associated with the polyphase matrix of the first $r$ rows will be decreased by one. This method is not restricted by the length of the filter, and we need not factorize the polyphase matrix into a special case. By unitary transform of two-dimensional CQF, we provide a parameterization method for constructing two-dimensional orthogonal wavelet filters. The choice of the parameters is not restricted by any implicit condition. Two-dimensional orthogonal wavelet filters can be chosen adaptively.

Methods for constructing non-separable orthogonal wavelets are developed. By the introduce of permutation of two-dimensional CQF, a method is presented for constructing two-dimensional non-separable orthogonal wavelets. Our construction begins with one-dimensional wavelet filters as in the construction of separable wavelet filters, but non-separable wavelet filters can be achieved. The lowpass & highpass

wavelet filters are given in explicit expressions. An algorithm is also provided for constructing Belogay-type wavelets. These wavelets are based on the commonly used dilation matrix $2I$. Their Regularity is discussed.

The problem of matrix extension related to the construction of orthogonal wavelets from interpolatory functions is well solved. For an $m$-band ($m \in Z,\ m > 2$) orthogonal interpolatory function, we provide the formulas for constructing the associated wavelet masks. For a pair of two-dimensional dual refinable functions, when one of them is interpolatory, the formulas for constructing the associated biorthogonal wavelet masks are also given.

A method is provided for constructing two-dimensional biorthogonal wavelets from a pair of dual refinable functions. If one of the dual refinable functions is supported in $[0,3] \times [0,3] \cap Z^2$, formulas are given for constructing the corresponding biorthogonal multiwavelet masks. If all the dual refinable functions are not supported in $[0,3] \times [0,3] \cap Z^2$, we shorten the support of the dual refinable functions to $[0,3] \times [0,3] \cap Z^2$ by increasing the multiplicity. Hence, in our method, the matrix extension by the Quillen-Suslin Theorem is avoided.

We describe a blind watermarking scheme for still image based on discrete non-separable wavelet transform (DNWT). Pseudo-random codes will be added to more coefficients in the high frequency sub-bands by DNWT than by discrete non-separable wavelet transform (DSWT). It is shown that the DNWT watermarking scheme is robust to some distortions such as noising, JPEG compression, cropping, and especially for sharpening. Furthermore, it is also shown that the DNWT watermarking scheme can not be substituted by adjusting the threshold such that the number of the DSWT coefficients to embed watermark is no less than the number of the DNWT coefficients.

Finally, based on parameterization of two-dimensional wavelet filters, we describe a blind watermarking system for ownership verification of digital images. The ample choice of wavelet filters will increase the difficulty for counterfeiters to gain the exact knowledge of our watermark. In this system, watermarks are inserted into several middle-frequency sub-bands. The existence of the watermark is asserted if any one of the correlation values is greater than a pre-determined threshold.

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