

DOCTORAL THESIS

Local absorbing boundary conditions for Korteweg-de-Vries-type equations

Zhang, Wei

Date of Award:
2014

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Abstract

The physicists and mathematicians have put a lot of efforts in the numerical analysis of various types of partial differential equations on unbounded domain. The time-dependent partial differential equations(PDEs) also have a wide range of applications in physics, geography and many other interdisciplines. This thesis is concerned with the numerical solutions of such kind of partial differential equations on unbounded spatial domain, especially the Korteweg-de Vries(KdV) equations. Since it is unable to solve the problem directly due to its unboundedness, the common way to surpass such difficulty is to introduce proper conditions on the truncated artificial boundaries and to approximate the problem on a bounded domain, which is also known as the Absorbing Boundary Conditions(ABCs).

One of the main contributions of this thesis is to design accurate local absorbing boundary conditions for linearized KdV equations and to extend the method to nonlinear KdV equations on unbounded domain. Padé approximation is the main tool to approximate the cubic root in the construction of local absorbing boundary conditions(LABCs) for a linearized KdV equation on unbounded domain. Besides, we also introduce the continued fraction method in the approximation of cubic root. To avoid the high-order derivatives in the absorbing boundary conditions, a sequence of auxiliary variables are applied accordingly. Then the original problem on unbounded domain is reduced to an approximated initial boundary value(IBV) problem defined on a finite domain.

Based on previous work, we are able to extend the method to the design of efficient local absorbing boundary conditions for nonlinear KdV equations on unbounded domain. The unifying approach method is applied to this nonlinear case. The idea of the unifying approach method is to separate inward- and outward-going waves and to build suitable approximated linear operator with a “one-way operator”. Then we unite the approximated linear operator with the nonlinear subproblem and propose boundary conditions for the nonlinear subproblem along the artificial boundaries. The numerical simulations are given to demonstrate the effectiveness and accuracy

of our local absorbing boundary conditions.

Keywords: Korteweg-de Vries equation; Local absorbing boundary conditions; Padé approximation; Continued fraction method; Unifying approach.

Acknowledgements

First and foremost, I want to take this opportunity to thank my supervisor, Prof. Wu Xiaonan, for the patient guidance, constant encouragement that he has provided throughout my time as his student. His profound knowledge in mathematics, modesty and prudence attitudes in research influence me a lot. This thesis could not have reached its present form, without his consistent guidance and illuminating advices. His professional quality and good habits in research, remarkable characteristics in behaviors, will benefit me for life. I also thank to my co-supervisor, Dr. Zeng Tiejong. The enthusiasm he has for his research is contagious and motivational for me during my Ph.D. pursuit.

I thank Dr. Li Hongwei for his generous help during my freshman year in getting into the field, and good collaboration in the next few years. I am also sincerely grateful to Mr. Yang Jiang and Dr. Zhang Jiwei for their selfless assistances and inspiring discussions in my study. I thank all the fellow students for their friendships. My dear roommates and friends have made my life in Hong Kong a memorable one. I thank them for all the precious moments.

I wish to thank Hong Kong Baptist University for providing quality courses and the whole person education. I am also greatly indebted to the teachers, secretaries and technicians at the Department of Mathematics for their continuous help.

Most of all, I wish to express my gratitude to my family, for their love and confidence in me all along.

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