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DOCTORAL THESIS

Meshless algorithm for partial differential equations on open and singular surfaces

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Abstract

Radial Basis function (RBF) method for solving partial differential equation (PDE) has a lot of applications in many areas. One of the advantages of RBF method is meshless. The cost of mesh generation can be reduced by playing with scattered data. It can also allow adaptivity to solve some problems with special feature. In this thesis, RBF method will be considered to solve several problems. Firstly, we solve the PDEs on surface with singularity (folded surface) by a localized method. The localized method is a generalization of finite difference method. A priori error estimate for the discreitzation of Laplace operator is given for points selection. A stable solver (RBF-QR) is used to avoid ill-conditioning for the numerical simulation.

Secondly, a H^2 convergence study for the least-squares kernel collocation method, a.k.a. least-square Kansa's method will be discussed. This chapter can be separated into two main parts: constraint least-square method and weighted least-square method. For both methods, stability and consistency analysis are considered. Error estimate for both methods are also provided. For the case of weighted least-square Kansa's method, we figured out a suitable weighting for optimal error estimation.

In Chapter two, we solve partial differential equation on smooth surface by an embedding method in the embedding space $\mathbb{R}^d$. Therefore, one can apply any numerical method in $\mathbb{R}^d$ to solve the embedding problem. Thus, as an application of previous result, we solve embedding problem by least-squares kernel collocation. Moreover, we propose a new embedding condition in this chapter which has high order of convergence. As a result, we solve partial differential equation on smooth surface with a high order kernel collocation method. Similar to chapter two, we also provide error estimate for the numerical solution. Some applications such as pattern formation in the Brusselator system and excitable media in FitzHughNagumo model are also studied.

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