DOCTORAL THESIS

The bandwidth and coloring problems of graphs

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The Bandwidth and Coloring Problems of Graphs

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Abstract

The bandwidth and cyclic bandwidth problems for a graph G can be viewed as embeddings (layouts) of G into a path and a cycle respectively. We first give a graph G with p vertices and a bijection $f : V(G) \rightarrow \{1, 2, \ldots, p\}$, then the bandwidth of G is defined by $B(G) = \min_{f \in F} B(G, f)$ where F is the set of possible bijections of G and $B(G, f) = \max_{uv \in E(G)} |f(u) - f(v)|$; the cyclic bandwidth of G is defined by $B_c(G) = \min_{f \in F} B_c(G, f)$ where F is the set of possible bijections of G, $B_c(G, f) = \max_{uv \in E(G)} \|f(u) - f(v)\|_c$ and $\|x\|_c = \min\{|x|, n - |x|\}$.

As the bandwidth problem was shown to be NP-complete by Papadimitriou [53] in 1976, researchers focused on acquiring the upper and lower bounds for the bandwidth of some special classes of graphs – grids [15], triangulated triangles [29], convex triangulation meshes [37], complete m-ary trees [61] and so on. And they discovered some theorems of bandwidth in terms of degrees of the vertices, maximum degree [14, 39], boundaries [24] and Diameters [14, 16, 18]. They also got some results on Trees [17, 65]. All these results are summarized in Chapter 2.

In Chapter 3, I will introduce the relative numbering algorithm that we obtained in 1997 for showing the bandwidth and cyclic bandwidth of a tree being identical. Following this idea, in Chapter 4, we shall give the conditions for characterizing graphs with equal bandwidth and cyclic bandwidth. With these conditions, we shall illustrate that each of the graphs in the special classes such as mn-grids $P_m \times P_n$ with $n \geq m \geq 3$, convex triangulation meshes $T_{m,n,l}$ with $\min\{m, n, l\} \geq 4$ is has identical bandwidth and cyclic bandwidth.

Besides, researchers also worked for the relations of bandwidths of similar graphs. They would like to investigate the changes of the bandwidth of a graph while local operations being carried out (see Section 2.1.5). In 1995, Wang and Yao [66] obtained the result on the maximum bandwidth under edge addition. The sharp upper bound of the cyclic bandwidth of a graph under the same operation will also be given. The result is shown in Chapter 5.

Another famous labelling problem in graph theory is the coloring problem. Since the *Four Coloring Problem* had been first posed in 1852 by Francis Guthrie, almost every mathematician attempted this problem but no one got an analytical proof of the problem in the traditional sense. In the past 150 years, the problem has been developed into several related coloring problems.

In this thesis, we discuss the edge-face total coloring problem on Halin graphs. The edge-face total chromatic number of 3-regular Halin graphs will be shown to be 4 or 5 in Section 7.1. The necessary and sufficient condition to characterize a 3-regular Halin graph of edge-face total chromatic number 4 or 5 will be provided in Section 7.2. In the last section of Chapter 7, we shall give the edge-face total chromatic number of Halin graphs with $\Delta \geq 4$.

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