

## MASTER'S THESIS

### Quasi-Monte Carlo sampling for computing the trace of a function of a matrix

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**Quasi-Monte Carlo Sampling for Computing the  
Trace of a Function of a Matrix**

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for the degree of  
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## Abstract

For a very large dimensional  $n \times n$  matrix  $\mathbf{A}$ , sometimes one needs to find the trace of its inverse or its determinant efficiently. This thesis explores the effectiveness of using quasi-Monte Carlo samplings to solve these problems. The methods for quasi-Monte Carlo (QMC), which are Hadamard-like (HL) samplings, will be applied to computing the the trace of the inverse or the determinant of the matrix. Comparisons to Monte Carlo (MC) sampling will be made. The various types of error analyses including worst case, random case and average case will also be discussed. Besides, we include the theories of control variate method and preconditioning technique in order to improve the accuracy and convergence rate of the MC and QMC samplings. Some numerical results for estimating the trace of the inverse and the determinant of a matrix by using the above techniques are collected. Finally, a brief conclusion is provided.

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