

DOCTORAL THESIS

Numerical solutions for the Navier-Stokes equations and the Fokker-Planck equations using spectral methods

Fok, Chin Man

Date of Award:
2002

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

**Numerical Solutions for the Navier-Stokes
Equations and the Fokker-Planck Equations using
Spectral Methods**

FOK Chin Man

A thesis submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

Principal Supervisor: Prof. TANG Tao

Hong Kong Baptist University

August 2002

Abstract

The main purpose of this thesis is to develop an efficient numerical methods for the computation of viscous incompressible flows as described by the Navier-Stokes equations. The emphasis is on spectral methods coupled with a second order semi-implicit time stepping procedure. The second topic of this thesis is concerned with a Hermite spectral approach for the Fokker-Planck equations which are defined in an infinity domain. The emphasis is to establish a convergence theory for the spectral method approach.

In chapter 1 a set of governing equations for incompressible flows is derived from invoking the physical laws of conservation. The conservation of mass and momentum leads to the Navier-Stokes equations.

In chapter 2 a second order finite difference method based on the streamfunction-vorticity formulation of the Navier-Stokes equations is presented. Numerical boundary conditions for the vorticity are discussed. Runge Kutta time stepping and its stability are also investigated. The numerical scheme is suited for the computation of both low and high Reynolds number flows. The resulting method performs well when used to compute the regularized cavity flow.

Chapter 3 provides the background of the spectral methods, including relevant theories in the Sobolev space and some computational aspects. Some results on the Hermite spectral approach appear to be original.

In chapter 4 an accurate scheme for the time-dependent incompressible Navier-

Stokes equations in terms of the primitive variables on a non-staggered grid is proposed. The novelty of the scheme lies primarily in a simple, consistent and accurate numerical approximation of the Neumann boundary condition for the pressure Poisson equation. Its use avoids the need for both fractional-step time discretization and staggered grids traditionally required for the most popular numerical methods based on the primitive variable formulation. In addition, the proposed scheme is coupled with spectral-Galerkin and semi-implicit time stepping methods. Excellent results are obtained by applying the scheme to 2D test problems with various values of the Reynolds number. A spectral accuracy is observed in space, and the unconditional stability of the scheme is verified numerically.

In chapter 5 the proposed scheme in Chapter 4 is applied to two test problems. The first one is a regularized cavity flow and the other one is the impulsively started flow around a cylinder.

In chapter 6 the convergence theory of a class for combined spectral-finite difference methods applied to the Fokker-Planck equation is established. It is shown that the Hermite based spectral methods are convergent with spectral accuracy in some weighted Sobolev space. The spectral convergence rate is demonstrated by a number of numerical experiments. A velocity scaling factor is used in the Hermite basis function and is shown to greatly improve the accuracy and effectiveness of the Hermite spectral approximation, with no increase in workload. The results based on this chapter will be published by *Mathematics of Computation*. Its electronic version in *Mathematics of Computation* can be found in www.ams.org/jourcgi/jour-getitem?pii=S0025-5718-01-01365-5.

Contents

Declaration	i
Abstract	iii
Acknowledgements	iv
Table of Contents	v
List of Tables	ix
List of Figures	xiii
Chapter 1 Introduction	1
1.1 Basic Conservation Laws	1
1.2 Similitude	6

1.3	Stream function - Vorticity Formulation	8
Chapter 2 Reynolds Number Boundary Condition		11
2.1	Description of the Scheme	12
2.2	Implementing Slip Boundary Condition	13
2.3	Temporal Discretization Methods	13
2.4	Numerical Test	17
Chapter 3 Spectral Method		24
3.1	Overview	26
3.2	Orthogonal Functions in Sobolev Spaces	26
3.3	Pseudospectral method	40
3.4	Differentiation matrices for polynomial basis function	43
3.5	Pseudospectral method for two-point BVPs	45
3.6	Spectral-Galerkin Method	48
3.7	2D Non-homogeneous Neumann Problem	57
3.8	2D Mixed Boundary Problem	61

Chapter 4 An Accurate Pressure Poisson Solver Scheme	65
4.1 Formulation	66
4.2 Outline of APPS scheme	67
4.3 Semi-implicit second-order APPS scheme	68
Chapter 5 Numerical Experiments for 2-D NSE	80
5.1 A Transformation	80
5.2 Treatment of the non-homogeneous Dirichlet boundary problem	81
5.3 A Driven Cavity Problem	82
5.4 Impulsively started flow around a cylinder	90
Chapter 6 Spectral-finite difference method for the Fokker-Planck equations	104
6.1 Background of the Fokker-Planck equation	105
6.2 Hermite spectral-finite difference method	110
Chapter 7 Conclusion	145
Bibliography	148

