

## MASTER'S THESIS

# Graph approach modeling and optimal heuristics for the one-dimensional cutting and packing problems

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*Date of Award:*  
2002

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Graph Approach Modeling and Optimal Heuristics  
for the One-Dimensional Cutting and Packing  
Problems

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A thesis submitted in partial fulfillment of the requirements  
for the degree of  
Master of Philosophy

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September 2002

# Abstract

We explore a model for one-dimensional cutting and packing problem, which is based on the Gilmore and Gomory's approach. We improve on the approach in order to address the growth in the number of cutting patterns. In our model, new sets of variables are formed by using the digraph representation. Incidence matrix of digraph is then applied to formulate the configuration of the incremental patterns with side constraints. In addition, we modify our model to handle the generalized problem with different stock / bin sizes, prices and supplies considerations. A Strategy for reducing the variables and constraints in the programming is suggested when the problem size is huge. Computational results of the test cases obtained from the OR-Library and other resources will be reported. The results demonstrate the significant contributions of our model by the leveling off of solution times with the increase in problem size. We also introduce one optimal heuristics algorithm. While this is intuitively obvious, we provide a logical proof for completeness and specify the optimality conditions. This algorithm can be used for pre-processing of the packing before the formulation of our model. As a result, the number of cutting patterns can be reduced and solution times can be improved. Furthermore, we modify our model to handle different types of Knapsack Problem. We attempt to explore the applicability of our approach/formulation to a different type of problem that has similar combinatorial requirements.

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