DOCTORAL THESIS

Determination of random schrödinger operators

Ma, Shiqi

Date of Award:
2019

Abstract

Inverse problems arise in many fields such as radar imaging, medical imaging and geophysics. It draws much attention in both mathematical communities and industrial members. Mathematically speaking, many inverse problems can be formulated by one or several physical equations and mathematical models. For example, the signal used in radar imaging is governed by Maxwell's equation, and most of geophysical studies can be formulated using elastic equation. Therefore, rigorous mathematical theories can be applied to study the inverse problems coming from this complex world.

Random inverse problem is a fascinating area studying how to extract useful statistical information from unknown object coming from real world. In this thesis, we focus on the study of inverse problem related to random Schrödinger operators. We are particularly interested in the case where both the source and the potential of the Schrödinger system are random.

In our first topic, we are concerned with the direct and inverse scattering problems associated with a time-harmonic random Schrödinger equation with unknown random source and unknown potential. The well-posedness of the direct scattering problem is first established. Three uniqueness results are then obtained for the corresponding inverse problems in determining the variance of the source, the potential and the expectation of the source, respectively, by the associated far-field measurements. First, a single realization of the passive scattering measurement can uniquely recover the variance of the source without the a priori knowledge of the other unknowns. Second, if active scattering measurement can be further obtained, a single realization can uniquely recover the potential function without knowing the source. Finally, both the potential and the first two statistic moments of the random source can be uniquely recovered with full measurement data.

Our second topic also focuses on the case where only the source is random. But in the second topic, the random model is different from our first topic. The second random model has received intensive study in recent years due to the reason that this random model has more flexibility fitting with different regularities. The recovering

framework is similar to our first topic, but we shall develop different asymptotic estimates of the higher order terms, which is more difficult than the first one.

Lastly, based on the previous two results, we study the case where both the source and the potential are random and unknown. The ergodicity is used to establish the single realization recovery. The asymptotic estimates of higher order terms are based on pseudodifferential operators and microlocal analysis.

Three major novelties of our works in this thesis are that, first, we studied the case where both the source and the potential are unknown; second, both passive and active scattering measurements are used for the recovery in different scenarios; finally, only a single realization of the random sample is required to establish the recovery of useful information.

Table of Contents