

## MASTER'S THESIS

### A class of operator splitting methods for least absolute shrinkage and selection operator (LASSO) models

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**A Class of Operator Splitting Methods for Least  
Absolute Shrinkage and Selection Operator  
(LASSO) Models**

**MO Lili**

A thesis submitted in partial fulfillment of the requirements  
for the degree of  
Master of Philosophy

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# Abstract

Sparsity of statistical models is important to interpret high dimensional data. Linear regression models with sparsity have various applications in many real-world processes, including signal processing, image processing, gene selection and comparative genomic hybridization. Commonly, it involves heavy computation for combinatorial search for finding such models with sparsity by model selection. The least absolute shrinkage and selection operator (LASSO) is a popular technique for model selection and estimation in linear regression to induce sparsity.

In this thesis, we study several general LASSO models, including group LASSO, sparse group LASSO and fused LASSO. We focus on the applications of alternating direction method (ADM) of multipliers to these LASSO models, including the applications of ADM to LASSO and group LASSO, and a linearized version of ADM to sparse group LASSO and fused LASSO. This approach can solve large scale problems efficiently. Some numerical study for colon cancer and leukemia cancer data sets are reported.

**Keywords:** Sparse model, Linear regression, LASSO, Alternating direction method of multipliers, Variable selection, Linearization

# Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	vii
List of Figures	viii
List of Abbreviations	ix
Chapter 1 Introduction	1
Chapter 2 LASSO Models	5
2.1 LASSO model . . . . .	5
2.2 Generalized LASSO models . . . . .	6
2.2.1 Group LASSO . . . . .	6
2.2.2 Sparse group LASSO . . . . .	8
2.2.3 Fused LASSO . . . . .	9
Chapter 3 Some Operator Splitting Algorithms for LASSO Models	10
3.1 Applications of alternating direction method of multipliers to LASSO and group LASSO . . . . .	10
3.1.1 Application of alternating direction method of multipliers to LASSO . . . . .	10

3.1.2	Application of alternating direction method of multipliers to group LASSO . . . . .	13
3.1.3	Convergence analysis . . . . .	14
3.2	Applications of linearized alternating direction method of multipliers to sparse group LASSO and fused LASSO . . . . .	15
3.2.1	Application of linearized alternating direction method of multipliers to sparse group LASSO . . . . .	15
3.2.2	Application of linearized alternating direction method of multipliers to fused LASSO . . . . .	17
3.2.3	Convergence analysis . . . . .	18
Chapter 4	Numerical Experiments	25
4.1	Implementation details . . . . .	26
4.2	Alternating direction method of multipliers for LASSO . . . . .	26
4.2.1	Simulation data . . . . .	26
4.2.2	Colon tumor data . . . . .	28
4.2.3	Leukaemia cancer data . . . . .	32
4.3	Alternating direction method of multipliers for group LASSO . . . . .	33
4.3.1	Simulation data . . . . .	33
4.3.2	Colon tumor data . . . . .	34
4.3.3	Leukaemia cancer data . . . . .	37
4.4	Linearized alternating direction method of multipliers for sparse group LASSO . . . . .	39
4.4.1	Simulation data . . . . .	39
4.4.2	Colon tumor data . . . . .	40
4.4.3	Leukaemia cancer data . . . . .	42
4.5	Linearized alternating direction method of multipliers for fused LASSO	44
4.5.1	Simulation data . . . . .	44
4.5.2	Colon tumor data . . . . .	45
4.5.3	Leukaemia cancer data . . . . .	46
Chapter 5	Conclusions	49

References	55
Curriculum Vitae	56