

## MASTER'S THESIS

### Bandwidth problems of graphs

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# **Bandwidth Problems of Graphs**

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### Abstract

The bandwidth and cyclic bandwidth problems for a graph  $G$  can be viewed as embeddings (layouts) of  $G$  into a path and a cycle respectively. We first give a graph  $G$  with  $n$  vertices and a bijection  $f : V(G) \rightarrow \{1, 2, \dots, n\}$ , then the bandwidth of  $G$  is defined by  $B(G) = \min_{f \in F} B(G, f)$  where  $F$  is the set of possible bijections of  $G$  and  $B(G, f) = \max_{uv \in E(G)} |f(u) - f(v)|$ ; the cyclic bandwidth of  $G$  is defined by  $B_c(G) = \min_{f \in F} B_c(G, f)$  where  $F$  is the set of possible bijections of  $G$ ,  $B_c(G, f) = \max_{uv \in E(G)} |f(u) - f(v)|_c$  and  $|x|_c = \min\{|x|, n - |x|\}$ .

As the bandwidth problem shown to be *NP*-complete by Papadimitriou (1976), in this thesis, we shall focus on the upper and lower bounds for the bandwidth of some special classes of graphs — convex triangulation meshes, complete  $m$ -ary trees and three dimensional meshes, as well as the changes of the bandwidth of graphs under edge-addition. Then by showing that the bandwidth and cyclic bandwidth of some graphs, including trees, are equal, we shall conclude that the cyclic bandwidth problem of graphs, of trees and of caterpillars with hairs length at most 3 are also *NP*-complete. Lastly we shall as well go to investigate the changes of cyclic bandwidth of graphs with adding an edge.

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