

## MASTER'S THESIS

### Bandwidth problems of graphs

Chan, Wai Hong

*Date of Award:*  
1996

[Link to publication](#)

#### General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

# **Bandwidth Problems of Graphs**

**CHAN Wai Hong**

**A thesis submitted in partial fulfilment of the requirements**

**for the degree of**

**Master of Philosophy**

**September 1996**

**Hong Kong Baptist University**



844

8-9-97

1424603X

TH

M. PHIL

1996 CB

## Abstract

The bandwidth and cyclic bandwidth problems for a graph  $G$  can be viewed as embeddings (layouts) of  $G$  into a path and a cycle respectively. We first give a graph  $G$  with  $n$  vertices and a bijection  $f: V(G) \rightarrow \{1, 2, \dots, n\}$ , then the bandwidth of  $G$  is defined by  $B(G) = \min_{f \in F} B(G, f)$  where  $F$  is the set of possible bijections of  $G$  and  $B(G, f) = \max_{uv \in E(G)} |f(u) - f(v)|$ ; the cyclic bandwidth of  $G$  is defined by  $B_c(G) = \min_{f \in F} B_c(G, f)$  where  $F$  is the set of possible bijections of  $G$ ,  $B_c(G, f) = \max_{uv \in E(G)} |f(u) - f(v)|_c$  and  $|x|_c = \min\{|x|, n - |x|\}$ .

As the bandwidth problem shown to be *NP*-complete by Papadimitriou (1976), in this thesis, we shall focus on the upper and lower bounds for the bandwidth of some special classes of graphs — convex triangulation meshes, complete  $m$ -ary trees and three dimensional meshes, as well as the changes of the bandwidth of graphs under edge-addition. Then by showing that the bandwidth and cyclic bandwidth of some graphs, including trees, are equal, we shall conclude that the cyclic bandwidth problem of graphs, of trees and of caterpillars with hairs length at most 3 are also *NP*-complete. Lastly we shall as well go to investigate the changes of cyclic bandwidth of graphs with adding an edge.

# Table of Contents

**Abstract**    *i*

**Declaration**    *ii*

**Acknowledgments**    *iii*

**1. Introduction**    *1*

- 1.1 Background    *1*
- 1.2 Bandwidth of Graphs    *4*
- 1.3 Cyclic Bandwidth of Graphs    *13*

**2. Bandwidth of Convex Triangulation Meshes**    *16*

- 2.1 Introduction    *16*
- 2.2 Convex Triangulation Meshes    *17*
- 2.3 The Main Result    *21*

**3. Bandwidth of Complete  $m$ -ary Trees**    *25*

- 3.1 Introduction    *25*
- 3.2 Algorithm for Numbering Complete  $m$ -ary Trees by CHUNG and Counterexamples    *26*
- 3.3 A Rectified Algorithm and Vertices Reallocation Method    *32*

**4. Bandwidth of Three Dimensional Meshes**    *39*

- 4.1 Introduction    *39*
- 4.2 Upper Bound for The Bandwidth of  $TT_m$     *40*
- 4.3 "Level by Level" Numbering    *45*

<b>5.</b>	<b>Bandwidth of Graphs with Adding an Edge</b>	<b>47</b>
5.1	Introduction	47
5.2	Main Results	48
<b>6.</b>	<b>Cyclic Bandwidth of Graphs</b>	<b>58</b>
6.1	Introduction	58
6.2	Relative Numbering	59
6.3	Graphs with Cyclic Bandwidth Equal to Bandwidth	61
6.4	Other Results	65
<b>7.</b>	<b>Cyclic Bandwidth of Graphs with Adding an Edge</b>	<b>68</b>
7.1	Introduction	68
7.2	Main Results	68
<b>8.</b>	<b>Conclusion</b>	<b>75</b>
	<b>References</b>	<b>77</b>
	<b>VITA</b>	<b>81</b>