

DOCTORAL THESIS

Mathematical study on plasmon materials and their applications

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Abstract

This thesis is concerned with the mathematical analysis of plasmon materials and their applications, including the cloaking effect and the super-resolution in imaging induced by plasmon resonances. We consider these phenomena in two regimes, namely, the quasi-static regime and the finite-frequency regime beyond the quasi-static approximation.

In the quasi-static regime, we first show that the plasmon resonance could occur for the elastic system in both two and three dimensions. By establishing the primal and dual variational principles and constructing the perfect plasmon waves, we prove that the plasmon resonance can occur for a delicate plasmonic configuration with appropriately choosing Lamé parameters. We also apply the spectral method to show the phenomenon of cloaking due to anomalous localized resonance (CALR) through analyzing the spectral system of the Neumann-Poincaré (N-P) operator. Moreover, based on the obtained spectral properties of the N-P operator, we strictly verify the plasmon resonance in the quasi-static approximation and construct a general and novel class of plasmonic configurations to ensure the occurrence of the CALR, which significantly generalizes the existing research on plasmon resonances in the literature. Finally, we derive the perturbed displacement field associated with a given elastic source field with the presence of nanoparticles. By analyzing the spectral properties of the associated Neumann-Poincaré operator, the leading-order term of the perturbed elastic wave field is determined, which could help to achieve the super-resolution in the elastic imaging.

For the case beyond the quasi-static approximation, we develop two approaches to achieve the plasmon resonance. The first one is achieved by the explicit construction, though it is very delicate and subtle. We first show that the cloaking due to anomalous localized resonance could occur for the Helmholtz system within finite frequencies beyond the quasi-static approximation. More precisely, by investigating the spectral system of the corresponding Neumann-Poincaré operator within finite frequencies, we include the plasmon parameters, the shape of the plasmonic inclusion and the source term as a whole system to achieve the phenomenon of the cloaking due

to anomalous localized resonance. Furthermore, we show that the surface plasmon resonance and the cloaking effect can occur for the Maxwell system beyond the quasi-static approximation by calculating the spectral system of the matrix-valued integral operator. The other one is achieved via the localization and geometrization. Through the investigation on the eigenfunctions of the corresponding Neumann-Poincaré operator, we show that the plasmon resonance occurs locally near the high-curvature point of the plasmonic inclusion. It is worth mentioning that we present the first investigation in the literature on the geometric structures of the Neumann-Poincaré eigenfunctions.

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