

DOCTORAL THESIS

Some research on mixed finite element methods

Cheng, Xiao Liang

Date of Award:
1995

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

Some Research on Mixed Finite Element Methods

CHENG Xiao-Liang

A thesis submitted in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

September 1995

Hong Kong Baptist University



8/4
19-4-96
1361458/
TH

TH
PH.D
1995 CS

Abstract

The thesis consists of five chapters.

In the first chapter we discuss some finite element method for solving Timoshenko beam and circular arch problems. To avoid locking phenomenon, we use the reduced integration technique and add the bubble function space to rotation variable. The method for Timoshenko beam is aligned with Petrov-Galerkin formulation in [65] and can be applied to Reissner-Mindlin plate. Optimal error estimates are proved uniformly with respect to small parameter for our linear element scheme. A modification of Reddy's scheme [78] and numerical performance of circular arch problem are also presented.

In Chapter Two we apply the finite element methods to Reissner-Mindlin plate model. We propose a new linear scheme and prove that the approximate displacement and rotation converge at optimal rates uniformly with respect to thickness t . Our scheme is simpler than that in recent works [4,17,20,39,40].

In Chapter Three we discuss a finite element method related to the reduced integration technique for solving the biharmonic equation. On a rectangular mesh, a similar scheme is proposed in [68] and its convergence is analyzed in [62]. First we modify the scheme proposed in [68] and prove the optimal order error estimate without the extra smoothness assumption on the solution made in [62]. On a triangular mesh, an analogous scheme is proposed and its optimal order convergence property is proved. Some numerical results are given to show the convergence behavior of the numerical solutions.

In Chapter Four we discuss some mixed finite element methods related to the reduced integration penalty method for solving the Stokes problem. We prove optimal order error estimates for bilinear-constant and biquadratic-bilinear

velocity-pressure finite element solutions. Our theoretical results are consistent with the numerical results reported in [21,71]. Also we provide an interesting example to explain some relationship among meshes, checkerboard mode and inf-sup condition. We derive a new formulation of penalty method for solving the Stokes problem. The checkerboard mode of general $Q_n - Q_{n-1}$ velocity-pressure elements is also presented.

In the last chapter we present some preconditioning iterative algorithms to solve the linear system, which arises frequently from the mixed finite element formulations for the Stokes equations or other problems of elasticity. At the same time, the domain decomposition method for the Stokes equation based on the penalty formulation is also discussed.

Contents

Abstract	(i)
Declaration	(iii)
Acknowledgments	(iv)
Introduction	(1)
1. Finite element methods for Timoshenko beam and circular arch problems	(13)
1.1 Introduction	(13)
1.2 Timoshenko beam problem	(14)
1.2.1 Timoshenko beam model	(14)
1.2.2 Finite element approximation	(16)
1.2.3 Error estimate	(19)
1.2.4 Numerical results	(21)
1.3 Circular arch problem	(26)
2. Finite element methods for the Reissner–Mindlin plate problem	(30)
2.1 The Reissner-Mindlin plate model	(30)
2.2 A linear element scheme	(33)
2.3 A variant linear scheme	(38)
3. A finite element method related to reduced integration for biharmonic equation	(40)
3.1 Introduction	(40)
3.2 A modified scheme on rectangular mesh	(42)
3.3 A scheme on triangular mesh	(47)

3.4 Numerical experiments	(54)
4. Analysis of some mixed elements for the Stokes problem	(57)
4.1 Introduction.....	(57)
4.2 An abstract error estimate.....	(60)
4.3 Macro-element technique	(63)
4.4 Bilinear-constant velocity-pressure element	(65)
4.5 Biquadratic-bilinear velocity-pressure element	(69)
4.6 Numerical experiments	(74)
4.7 The checkerboard mode and inf-sup condition	(75)
4.7.1 The checkerboard mode	(76)
4.7.2 The inf-sup stable condition	(79)
4.7.3 A new penalty formulation	(82)
4.7.4 The $Q_n - Q_{n-1}$ rectangular elements.....	(83)
5. Some preconditioning iterative methods for solving the saddle point problems	(86)
5.1 Introduction.....	(86)
5.2 The preconditioning iterative method	(87)
5.3 The preconditioning conjugate gradient method	(91)
5.4 The minimum residual method.....	(94)
5.5 A DDM for Stokes equation using penalty method.....	(96)
Conclusions and Recommendations	(102)
Bibliography	(103)
Vita	(110)