

MASTER'S THESIS

Integral functional methods in stochastic filtering problems

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Integral Functional Methods
in
Stochastic Filtering Problems

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Integral Functional Methods in Stochastic Filtering Problems

by

Lam Wai Hung

Abstract

This thesis is divided into three parts. In the first part, filtering problems of the additive white Gaussian noise model are discussed. When the n th order Wiener-Hopf equation is solved, a suboptimal estimate can be expressed as a series of Wiener integrals with respect to the observation process. However, to solve the n th order Wiener-Hopf equation is a difficult task. Fortunately, this problem can be solved if the optimal estimate is approximated by a series of Fourier-Hermite sets. A product-to-sum formula of Hermite polynomials is used so that theory of Fourier-Hermite expansion of the optimal estimate can be simplified. The Fourier-Hermite coefficients of the estimate satisfy a system of linear equations. A recursive relation of the system of different order estimates is found. Moreover, a new robust method to evaluate the Fourier-Hermite sets is also given. In the second part, recursive filters for some particular systems are the point of discussion. The Wiener-Hopf technique in Kalman-Bucy filtering theory is extended. Recursive linear optimal filters of two nonlinear systems are realized in stochastic differential equations because the first order Wiener-Hopf equations of the nonlinear systems can be transformed to Riccati differential equations. By some modifications of these filters, recursive nonlinear filters of the systems can be obtained. It is well known that the optimal estimate can be expressed as a ratio of two infinite series of iterated Wiener integrals. The final part proves that kernels of the integrals of a class of diffusion processes can be found recursively.

Content

1.	Introduction	1
1.1	Historical background	1
1.2	Problem formulation	2
1.3	Optimality, recursiveness and robustness	4
1.4	The nonlinear filtering equations	6
2.	Suboptimal Solutions by Iterated Wiener Integrals	13
2.1	Introduction	13
2.2	Iterated Wiener integral	13
2.3	Derivation of the nth order Wiener-Hopf equation	16
2.4	The nth order Wiener-Hopf equation and Fredholm integral equations	23
3.	Approximation by a Series of Fourier-Hermite Sets	30
3.1	Introduction	30
3.2	Fourier-Hermite sets	31
3.3	Determination of Fourier-Hermite coefficients	34
3.4	Recursive methods to compute Fourier-Hermite sets	40
4.	Recursive Filters for Two Classes of Non-stationary Processes	46
4.1	Introduction	46
4.2	Recursive linear filter for nonlinear systems I	46
4.3	Recursive linear filter for nonlinear systems II	54
4.4	A recursive nonlinear filter obtained by the linearly optimal filtering theory	58
5.	Explicit Solutions for a Class of Nonlinear Filtering problems	62
5.1	Introduction	62
5.2	An explicit solution	64
6.	Conclusion	68
7.	Appendix	71
7.1	Appendix I Proof of lemma 3.2	71
7.2	Appendix II Proof of lemma 4.3	72
	References	73
	Vita	77