

## MASTER'S THESIS

### Full friendly index sets of cartesian product of two cycles

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# Full Friendly Index Sets of Cartesian Product of Two Cycles

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Master of Philosophy

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# Abstract

A friendly labeling of a graph  $G = (V, E)$  is an assignment  $f$  of integers, either 0 or 1, to the vertices of  $G$  with a restriction that the difference in the amount of vertices assigned between 0's and 1's is at most one. Under such a labeling, the edge labeling  $f^*$  is induced such that each edge  $xy$  of  $G$  is assigned to either 0 or 1 by  $f^*(xy) \equiv f(x) + f(y) \pmod{2}$ . Also, a subtraction of the amount of edges assigned to 0's from that assigned to 1's is called the friendly index of  $G$  under a friendly labeling. If an assignment to the vertices with an additional restriction that a friendly index is equal to -1,0 or 1, the labeling is not only called friendly labeling but also cordial labeling. Moreover, a full friendly index set of  $G$  is a set of all possibilities of friendly indices induced by assigning the integers to the vertices of  $G$ .

From the development of friendly labeling, we will establish a relationship between cordial labeling and friendly labeling as well as figure out the extreme friendly indices of Cartesian product of two cycles. Finally, we completely determine the full friendly index sets of different kinds of Cartesian product of two cycles in this thesis.

# Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Figures	vi
Chapter 1 Introduction	1
1.1 Motivation and Purpose . . . . .	2
1.2 Overview of The Thesis . . . . .	2
Chapter 2 Literature Review	3
2.1 Terminologies . . . . .	3
2.2 Historical Background and Previous Results of Full Friendly Index Sets . . . . .	4
2.2.1 Cordial Labeling . . . . .	4
2.2.2 Friendly Index Set . . . . .	8
2.2.3 Full Friendly Index Set . . . . .	10
Chapter 3 Extreme Friendly Index	13
3.1 The Upper Bounds . . . . .	14
3.2 The Lower Bounds . . . . .	17

Chapter 4	Full Friendly Index Set	23
4.1	General Properties of Cartesian Product of Two Cycles . . . . .	23
4.2	Cartesian Product of Two Odd Cycles . . . . .	25
4.3	Cartesian Product of A Larger Odd Cycle with An Even Cycle . . .	34
4.4	Cartesian Product of A Larger Even Cycle with An Odd Cycle . . .	40
4.5	Cartesian Product of Two Even Cycles . . . . .	50
Chapter 5	Concluding Remarks	62
5.1	Summary of Results . . . . .	62
5.2	Further Research . . . . .	63
	Bibliography	64
	Index	67
	Curriculum Vitae	69