

## MASTER'S THESIS

### The strong chromatic index of cubic Halin graphs

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# **The Strong Chromatic Index of Cubic Halin Graphs**

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Master of Philosophy**

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## Abstract

A strong  $k$ -edge-coloring of a graph  $G$  is an assignment of  $k$  colors to the edges of  $G$  in such a way that any two edges meeting at a common vertex, or being adjacent to the same edge of  $G$ , are assigned different colors. The strong chromatic index of  $G$  is the smallest number  $k$  for which  $G$  has a strong  $k$ -edge-coloring. A Halin graph is a planar graph consisting of a tree with no vertex of degree two and a cycle connecting the end vertices of the tree. A caterpillar is a tree such that the removal of the leaves becomes a path. Among all cubic Halin graphs, a necklace is a graph whose tree is a caterpillar of a particular type, and a complete Halin graph is a graph whose tree is a complete cubic tree, in which all non-leaf vertices have degree three and all leaves are at the same distance from the root vertex. In this thesis, we shall prove that the general bounds for the strong chromatic index of a cubic Halin graph are sharp, and study strong edge-colorings of cubic Halin graphs whose tree is a caterpillar, necklaces and complete cubic Halin graphs.

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