

DOCTORAL THESIS

Shape representation based on wavelet skeleton

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Shape Representation Based on Wavelet Skeleton

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for the degree of
Doctor of Philosophy

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Abstract

Representation of a shape using a suitable form is essential to subsequent recognition of the shape. An appropriate representation for shape is its skeleton. The objective of this study is to extract skeletons from planar shapes, which have the following properties: (1) conforming to human perceptions of the original shapes, (2) being centred inside the original shapes, (3) being efficiently computable, (4) being robust against noise and affine transformations.

Two wavelet-based approaches for computing skeleton of the shape are developed. The first one depends on the development of modulus maxima symmetry of wavelet transform. A key issue in skeleton computation is the identification of local symmetries. Local symmetries are conventionally defined in the continuous domain, and they are difficult to be computed in the discrete domain. To solve this problem, a new concept called Modulus Maxima Symmetry of Wavelet Transform is proposed. Where a novel wavelet function plays a key role in this scheme in that it possesses the three significant characteristics better suited for describing the edge information of the shape than many commonly used filters, namely: (1) The position of the local modulus maximum of the wavelet transform with respect to the ribbon-like shape is independent of the gray-levels of the image; (2) When the appropriate scale of the wavelet transform is selected, the local modulus maxima of the wavelet transform of the ribbon-like shape produce two new parallel contours, which are located symmetrically at two sides of the original one and have the same topological and geometric properties as that of the original shape; (3) The distance between these two parallel contours equals to the scale of the wavelet transform, which is independent of the width of the shape. This new scheme consists of two phases: (a) Generation of wavelet skeleton — Based on the desirable properties of the new wavelet function, symmetry analyses of the modulus maxima of the wavelet transform is described. Mid-points of all pairs of contour elements can be connected to generate a skeleton of the shape, which is defined as wavelet skeleton. (b) Modification of the primary wavelet skeleton — Thereafter, two sets of techniques which are suitable for skeletonization of different structure shapes are utilized for modifying and amending the

artifacts of the primary wavelet skeleton.

Another proposed approach for computing skeletons is based on the local minima of wavelet transform moduli. The principal concept of the method is inspired by the previous desirable properties of the local minima of wavelet transform moduli. It provides simple and direct strategies for computing skeleton of the ribbon-like shape, and are well applicable to the ribbon-like shape with all sorts of complicated width structures, as the modulus-maximum-based algorithm fails to do. In particular, its scale of the WT in practice can be free selected instead of manual selection according the width of the shape in the modulus-maximum-based algorithm.

Two approaches not only makes great improvements on computing primary skeletons and removing artifacts, but also are applicable to the general gray image, as most existing methods fail to process. Meanwhile, the representation of the skeleton is robust against noise and affine transformation. The final skeletons obtained from the proposed methods possess a number of desirable properties including planarity, homotopy, preservation of geometrical properties, centrality, computability, efficiency and robustness.

The experiments of the algorithms applied to various shapes are conducted, and the promising results are yielded. Experimental results show that the proposed schemes are superior to existing skeletonization methods in terms of computational efficiency, geometrical properties, robustness and applicabilities.

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